CLASS 12 - MATHEMATICS FORMULA BOOK FOR CBSE BOARD

RELATIONS & FUNCTIONS

- **Relations**
  Cartesian Product of Sets
  $A \times B = \{(x, y) : x \in A, y \in B\}$
- **Functions**
  A function $f : X \to Y$ is **one-one (injection)** if $f(x) = f(y) \Rightarrow x = y$ or $x \neq y \Rightarrow f(x) \neq f(y)$
  A function $f : X \to Y$ is **onto (surjection)** if to each $y \in Y$, there exists $x \in X$ such that $f(x) = y$.
  f is **invertible** $\Leftrightarrow$ f is one-one onto **(Bijection)**
  The **number of functions** from a finite set A to a finite set $B = (n(B))^{n(A)}$
  The **number of one-one functions** that can be defined from a finite set A to a finite set B is $^{n(B)}P_{n(A)}$ if $n(B) \geq n(A)$ and 0, otherwise.
- If a relation is reflexive, symmetric and transitive then the relation is an equivalence relation.

INVERSE TRIGONOMETRY FUNCTIONS

- **Properties of Inverse Trigonometric Functions**

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, |x| \leq 1$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, x \in R$$

$$\sec^{-1} x + \text{cosec}^{-1} x = \frac{\pi}{2}, |x| \geq 1$$

$$\sin^{-1}(-x) = -\sin^{-1} x, |x| \leq 1$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x, |x| \leq 1$$

$$\tan^{-1}(-x) = -\tan^{-1} x, x \in R$$

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy < 1$$

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right), xy > -1$$

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1} x, |x| \leq 1$$

$$\cos^{-1}\left(\frac{1+x^2}{1-x^2}\right) = 2\tan^{-1} x, x \geq 0$$

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1} x, |x| < 1$$

$$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right),$$
$x \geq -1, y \leq 1$

$$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right),$$
$x \geq -1, y \leq 1$

MATRICES AND DETERMINANTS

- **Transpose of a Matrix**
  Let $A = [a_{ij}]_{m \times n}$ matrix. Then the transpose of A, denoted by A^T or A', is an $n \times m$ matrix such that $A^T = [a_{ji}]_{n \times m}$ $\forall$ $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.
  Thus, A^T is obtained from A by changing its rows into columns and its columns into rows.
- **Properties of Transpose**
  Let A and B be two matrices. Then

(i) $(A^T)^T = A$
(ii) $(A + B)^T = A^T + B^T$, where A and B being of the same order.
(iii) $(kA)^T = kA^T$, k be any scalar (real or complex)
(iv) $(AB)^T = B^T A^T$, A and B being conformable for the product AB. (Reversal law).
(v) $(ABC)^T = C^T B^T A^T$.

- **Some Special Matrices**
  Symmetric matrix : A square matrix $A = [a_{ij}]$ is called a symmetric matrix if $a_{ij} = a_{ji}$ for all i, j.
  Skew-symmetric matrix : A square matrix $A = [a_{ij}]$ is a skew-symmetric matrix if $a_{ij} = -a_{ji}$ for all i, j.
  Orthogonal matrix : A square matrix A is called an orthogonal matrix if $AA^T = A^T A = I$.

Equivalent matrices : Two matrices A and B are equivalent if one can be obtained from the other by a sequence of elementary row transformations.

- **Invertible Matrices**

  If A is a square matrix of order n and if there exists another square matrix B of the same order n, such that $AB = BA = I$, then B is called the inverse matrix of A and it is denoted by A^{-1}. In that case A is said to be invertible.

- **Uniqueness of inverse**

  Inverse of a square matrix, if it exists, is unique.

  If A and B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1} A^{-1}$.

- **Inverse of a Matrix by Elementary Operations**

  To find A^{-1} using elementary row operations,write $A = IA$ and apply a sequence of row (column) operations on $A = IA$ till we get, $I = BA$. The matrix B will be the inverse of A.

- **Properties of Determinant**

(i) det $I_n = 1$, where I_n is unit/identity matrix of order n.

(ii) det $O_n = 0$, O is null matrix (square matrix of any order).

(iii) $A = (a_{ij})_{n \times n}$ then $|A| = |A'|$ {Reflection Property}.

(iv) det (AB) = det A·det B, where A & B are matrices of same order.

(v) det $(kA) = k^n$ det A, if A is of order $n \times n$.

(vi) det (A^n) = (det A) if $n \in I^+$.

(vii) $|A| = 0$ iff

  (a) Any two rows or columns are identical.

  (b) Any two rows or columns are in proportion.

  (c) Each element of any row/column is zero.

(viii) If each element of a row/column of a determinant is multiplied by k then value of new determinant is k times the original determinant.

(ix) Determinant of a diagonal matrix is the product of its diagonal elements.

(x) If two rows/columns of a determinant are interchanged, then the determinant retains its absolute value but its sign is changed.

(xi) If each element of a row/column of a determinant is expressed as a sum of two or more terms, then determinant can be expressed as the sum of two or more determinants.

(xii) If any row/column of a determinant, a multiple of another row/column is added, then the value of determinant does not change.

(xiii) The sum of product of the elements of any row/column of a determinant with cofactors of the corresponding elements of any other row/column is zero.

- **Application of Determinants**

  Area of triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

  If area is zero then three points are collinear.

- **Minors and Cofactors**

  Minor of a_{ij} in $|A|$: The minor of an element a_{ij} in $|A|$ is defined as the value of the determinant obtained by deleting the i^{th} row and j^{th} column of $|A|$, and it is denoted by M_{ij}.

  Cofactor of a_{ij} in $|A|$: The cofactor C_{ij} of an element a_{ij} is defined as $C_{ij} = (-1)^{i+j} M_{ij}$.

- **Adjoint of a Matrix**

  Adjoint of a matrix $A = (a_{ij})_{n \times n}$ is defined as adj $A = [C_{ji}]_{n \times n}$, where

  C_{ij} represents cofactor of a_{ij} in $|A|$.

- **Properties of Adjoint of a Matrix**

  For every square matrix

  $A(\text{adj } A) = (\text{adj } A)A = |A|\cdot I$.

  If $|A| = 0$ matrix A is called singular else non-singular.

  If A and B are non singular matrices of the same order, then AB and BA are also non singular matrices of same order.

  $(\text{adj } AB) = (\text{adj } B)\cdot(\text{adj } A)$

  $(\text{adj } A)' = \text{adj } A'$.

  Let A be $n \times n$ matrix, then

  (i) $|\text{adj}A| = |A|^{n-1}$

  (ii) $\text{adj}(\text{adj}A) = |A|^{n-2}A$

  (iii) $|\text{adj}(\text{adj}A)| = |A|^{(n-1)^2}$

- **Inverse of a Matrix**

  A non-zero matrix A of order n is said to be invertible if there exists a square matrix B of order n such that $AB = BA = I$. We say $A^{-1} = B$.

  A matrix A is invertible if $|A| \neq 0$.

  $A^{-1} = \frac{1}{|A|}\cdot(\text{adj } A)$.

  If $AB = AC \Rightarrow B = C$ if $|A| \neq 0$.

$(AB)^{-1} = B^{-1}A^{-1}$.

$(A^T)^{-1} = (A^{-1})^T$.

If A is an invertible symmetric matrix then A^{-1} is also symmetric.

- **Solution of System of Linear Equations**

  Let $AX = B$ be the given system of equations :

(i) If $|A| \neq 0$, the system is consistent and has one unique solution.

(ii) If $|A| = 0$ and $(\text{adj } A)B \neq 0$, then the system is inconsistent so it has no solution.

(iii) If $|A| = 0$ and $(\text{adj } A)B = 0$, then the system is consistent but has infinitely many solutions.

CONTINUITY AND DIFFERENTIABILITY

Continuity

Continuity of a Function	Definitions
1. At a point	$f(x)$ is continuous at $x = a$ if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$.
2. In an interval	(i) Open interval : $f(x)$ is continuous at every point of (a, b). (ii) Closed interval : $f(x)$ is continuous in (a, b) and right & left continuous at a & b.
Discontinuity of a Function	
1. At a point	If $f(x)$ is not continuous at $x = a$.
2. In an interval	If $f(x)$ is not continuous at atleast one point in an interval.
Types of Discontinuity	
1. Removable discontinuity	Either $f(a)$ does not exist or $f(x) \neq \lim_{x\to a} f(x)$
2. Non-removable discontinuity	(i) First kind: If $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$ (ii) Second kind: If $\lim_{x\to a^-} f(x)$ or $\lim_{x\to a^+} f(x)$ or both do not exist.

Differentiability

Differentiablity of a function	
1. At a point	$f(x)$ is differentiable at $x = c$ if $\lim_{\substack{x\to c-h \\ h\to 0}} \frac{f(c-h)-f(c)}{-h} = \lim_{\substack{x\to c+h \\ h\to 0}} \frac{f(c+h)-f(c)}{h}$ or $Lf'(c) = Rf'(c)$
2. In an interval	(i) Open interval: If $f(x)$ is differentiable at every point of (a, b).(ii) Closed interval: If $f(x)$ is differentiable in (a, b) and also at a and b.

Derivative

Derivative of a function	
1. Left Hand Derivative (L.H.D.)	$Lf'(x) = \lim_{h\to 0} \frac{f(x-h)-f(x)}{-h}$
2. Right Hand Derivative (R.H.D.)	$Rf'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, If $Lf'(x) = Rf'(x)$, then $f'(x)$ exists.
3. Properties of derivative	(i) $(u \pm v)' = u' \pm v'$ (ii) $(uv)' = u'v + uv'$ (Product rule) (iii) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$, $v \neq 0$ (Quotient rule)

Derivatives of Important Functions

Function	Derivative	Function	Derivative	Function	Derivative
x^n	nx^{n-1}	$\sin x$	$\cos x$	$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$	$\cot x$	$-\text{cosec}^2 x$	$\sec x$	$\sec x \tan x$
$\text{cosec } x$	$-\text{cosec } x \cot x$	e^{ax}	ae^{ax}	e^x	e^x
$\sin^{-1} x$	$\frac{1}{\sqrt{(1-x)^2}}, x \in (-1,1)$	$\text{cosec}^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$, $x \in R - [-1, 1]$	$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}, x \in (-1,1)$
$\tan^{-1} x$	$\frac{1}{1+x^2}$, $x \in (-\infty, \infty)$	$\sec^{-1} x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$, $x \in R - [-1, 1]$	$\cot^{-1} x$	$-\frac{1}{1+x^2}$, $x \in (-\infty, \infty)$
$\log_e x$	$\frac{1}{x}$	a^x	$a^x \log_e a$	$\log_a x$	$\frac{1}{x \log_e a}$

Some kind of derivatives

1. Composite function (Chain rule)	(i) Let $y = f(t)$ and $t = g(x)$. Then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$
	(ii) Let $y = f(t)$, $t = g(u)$ and $u = h(x)$. Then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{du} \times \frac{du}{dx}$
2. Parametric function	If $x = f(t)$ and $y = g(t)$. Then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}, f'(t) \neq 0$
3. Second order derivative	Let $y = f(x)$, then $\frac{dy}{dx} = f'(x)$. If $f'(x)$ is differentiable, then $\frac{d}{dx}\left(\frac{dy}{dx}\right) = f''(x)$ or $\frac{d^2y}{dx^2} = f''(x)$
4. Logarithmic	If $y = u^v$, u and v are functions of x, then $\log y = v \log u$. Differentiating w.r.t. x, we get $\frac{d}{dx}(u^v) = u^v\left[\frac{v}{u}\frac{du}{dx} + \log u \frac{dv}{dx}\right]$
5. Implicit function	Here we differentiate the given function using $\frac{d}{dx}(\phi(y)) = \frac{d\phi}{dy} \times \frac{dy}{dx}$

Mean Value Theorems

1.	Rolle's theorem	If $f(x)$ is defined on $[a, b]$ such that it is (i) continuous on $[a, b]$ (ii) differentiable on (a, b) and (iii) $f(a) = f(b)$, then there exists at least one $c \in (a, b)$ such that $f'(c) = 0$
	Geometrical meaning	The tangent at point c on the curve is parallel to x-axis.
2.	Lagrange's mean value theorem	If $f(x)$ is defined on $[a, b]$ such that it is (i) continuous on $[a, b]$ (ii) differentiable on (a, b) and (iii) $f(a) \neq f(b)$, then there exists at least one $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$
	Geometrical meaning	The tangent at point c on the curve $= f(x)$ is parallel to the chord joining end points of the curve.

APPLICATION OF DERIVATIVES

- **Rate of Change of Quantity**

  For $y = f(x)$, $\frac{dy}{dx}$ denotes rate of change of y w.r.t. x.

  If $x = f(t)$, $y = g(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ is rate of change of y w.r.t. x.

- **Tangents and Normals**

  The slope of the tangent to $y = f(x)$ at (x_1, y_1) is $\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$ or $f'(x_1, y_1)$ and slope of the normal is

  $$-\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}} \text{ or } -\frac{1}{f'(x_1, y_1)}$$

  The tangent equation at (x_1, y_1) is $y - y_1 = f'(x_1, y_1)(x - x_1)$ and normal equation at (x_1, y_1) is

  $$y - y_1 = -\frac{1}{f'(x_1, y_1)}(x - x_1)$$

  Slope of tangent $\frac{dy}{dx} = \tan\theta$, θ is angle made by tangent positive x-axis.

  Angle between two curves is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$, where m_1 and m_2 are slopes of tangents to two curve at point of intersection.

Approximations and Errors

If $y = f(x)$ then $\delta y = f'(x)\,\delta x$ where

$\delta y = f(x + \delta x) - f(x)$ and δx is error in x and δy is corresponding error in y.

(i) Absolute error : δx is absolute error is x.

(ii) Relative error : $\frac{\delta x}{x}$ is the relative error.

(iii) Percentage error : $\left(\frac{\delta x}{x} \times 100\right)$ is the percentage error.

Increasing and Decreasing Function

1. Without derivative test in (a, b)	(i) Increasing function: If $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$ for all $x_1, x_2 \in (a, b)$ (ii) Strictly increasing function: If $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in (a, b)$ (iii) Decreasing function: If $x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in (a, b)$ (iv) Strictly decreasing function: If $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in (a, b)$
2. First derivative test in (a, b)	(i) Increasing function: If $f'(x) \ge 0$ for all $x \in (a, b)$ (ii) Strictly increasing: If $f'(x) > 0$ for all $x \in (a, b)$ (iii) Decreasing: If $f'(x) \le 0$ for all $x \in (a, b)$ (iv) Strictly decreasing: If $f'(x) < 0$ for all $x \in (a, b)$
3. Critical or Stationary point	The value of x for which $f'(x) = 0$

Maxima and Minima

Maximum	$f(x)$ has maximum at $x = c$, if $f(c) \ge f(x)$ in neighbourhood of c.
Minimum	$f(x)$ has minimum at $x = c$, if $f(c) \le f(x)$ is neighbourhood of c.
Local maxima	If $f(x) < f(c)\ \forall\ x$ in the given interval.
Local minima	If $f(c) < f(x)\ \forall\ x$ in the given interval.
Absolute maxima	If $f(x) \le f(c)\ \forall\ x$ in domain of $f(x)$.
Absolute minima	If $f(x) \ge f(c)\ \forall\ x$ in domain of $f(x)$.

Without derivative test	
Maximum Minimum	If $f(c) > f(x)\ \forall\ x, c \in I$, then $f(x)$ has maximum value in I and c is point of maxima. If $f(c) < f(x)\ \forall\ x, c \in I$, then $f(x)$ has minimum value in I and c is point of minima.
First derivative test	
Local maximum Local minimum Point of inflection	If $f'(x)$ changes sign from + ve to – ve as x increases through c. If $f'(x)$ changes sign from – ve to + ve as x increases through c. If $f'(x)$ does not change sign as x increases through c.
Second derivative test	
Local maximum Local minimum	If $f'(c) = 0$ and $f''(c) < 0$, then $f(x)$ has local maxima at c. If $f'(c) = 0$ and $f''(c) > 0$, then $f(x)$ has local minima at c.

INTEGRALS & DIFFERENTIAL EQUATIONS

- **Some Fundamental Integrals**

(i) $\int x^n \cdot dx = \frac{x^{n+1}}{n+1} + c$, where $n \neq -1$

(ii) $\int \frac{1}{x} dx = \log_e |x| + c$, where $x \neq 0$

(iii) $\int e^x\, dx = e^x + c$

(iv) $\int a^x\, dx = \frac{a^x}{\log_e a} + c$, where $a > 0$

(v) $\int \sin x\, dx = -\cos x + c$

(vi) $\int \cos x\, dx = \sin x + c$

(vii) $\int \sec^2 x\, dx = \tan x + c$

(viii) $\int \text{cosec}^2 x\, dx = -\cot x + c$

(ix) $\int \sec x \cdot \tan x\, dx = \sec x + c$

(x) $\int \text{cosec}\, x \cdot \cot x\, dx = -\text{cosec}\, x + c$

(xi) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$ or $-\cos^{-1} x + c$, where $|x| < 1$

(xii) $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$ or $-\cot^{-1} x + c$

(xiii) $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$ or $-\text{cosec}^{-1} x + c$

- **Some standard integrals using the above relations are shown below :**

(i) $\int \tan x\, dx = \log|\sec x| + c = -\log|\cos x| + c$

(ii) $\int \cot x\, dx = -\log|\text{cosec}\, x| + c = \log|\sin x| + c$

(iii) $\int \sec x\, dx = \log|\sec x + \tan x| + c$

(iv) $\int \text{cosec}\, x\, dx = \log|\text{cosec}\, x - \cot x| + c = \log\left|\tan\frac{x}{2}\right| + c$

- **Some special integrals**

(i) $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

(ii) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

(iii) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

(iv) $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log\left|x + \sqrt{x^2 \pm a^2}\right| + c$

(v) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c, |x| < |a|$

(vi) $\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$

- To evaluate integrals of type

(i) $\int \frac{lx+m}{ax^2+bx+c} dx$, where $l \neq 0, a \neq 0$

Take $lx + m = \frac{1}{2a}(2ax+b) + \left(m - \frac{lb}{2a}\right)$

(ii) $\int \frac{lx+m}{\sqrt{ax^2+bx+c}} dx$, where $l \neq 0, a \neq 0$

Put $z^2 = ax^2 + bx + c$,

so that $2z \cdot \frac{dz}{dx} = 2ax + b$ or $dz = \frac{(2ax+b)}{2z}dx$

(iii) $\int \frac{dx}{(lx+m)\sqrt{ax+b}}$, put $ax+b=z^2$

(iv) $\int \frac{dx}{(lx+m)\sqrt{ax^2+bx+c}}$, put $lx+m=\frac{1}{z}$

(v)

$\int \frac{dx}{(lx^2+m)\sqrt{ax^2+b}}$, put $\sqrt{ax^2+b}=xz$ or $x=\frac{1}{z}$

- **Integration by parts**

  $\int (\text{I} \cdot \text{II})\, dx = \text{I}\int \text{II}\, dx - \int\left[\frac{d}{dx}(\text{I})\int \text{II}\, dx\right]dx + c$

  Choice of I[st] function and II[nd] function depends on order of letters in the word ILATE

  I $\rightarrow$ Inverse function
  L $\rightarrow$ Logarithmic function
  A $\rightarrow$ Algebraic function
  T $\rightarrow$ Trigonometric function
  E $\rightarrow$ Exponential function

- A special integral

  $\int e^x\left[f(x)+f'(x)\right]dx = f(x)\cdot e^x + c$

- **Partial Fractions and their uses in integration.**

  If the integrand is a rational function, $\frac{p(x)}{q(x)}$

  (i) If degree $(p(x))$ < degree $(q(x))$

  i.e., $f(x)=\frac{mx+n}{(x-a)(x-b)}$, $a \neq b$ then we write

  $\frac{mx+n}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}$, A and B being constants.

  (ii) If degree $(p(x))$ = degree $(q(x))$ or degree $(p(x))$ > degree $(q(x))$ of non-repeated linear factors, *i.e.*, $f(x)=\frac{mx^2+nx+l}{(x-a)(x-b)}$, $a \neq b$ then we write $\frac{mx^2+nx+l}{(x-a)(x-b)}=m+\frac{A}{x-a}+\frac{B}{x-b}$

  (iii) If denominator $q(x)$ contains linear factors, some of which are repeated, *i.e.*, integrand is of the form $\frac{p(x)}{(x-a)(x-b)^2}$, then we write the integrand as $\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{(x-b)^2}$

- To evaluate integral of the type

  (i) $\int \frac{x^2+A}{x^4+kx^2+A^2}\,dx$

  Divide numerator and denominator by x^2 and substitute $x-\frac{A}{x}=u$, A being any positive constant.

  (ii) $\int \frac{x^2-A}{x^4+kx^2+A^2}\,dx$

  Divide numerator and denominator by x^2 and substitute $x+\frac{A}{x}=t$; A being positive constant.

  (iii) $\int \frac{ax^2+bx+c}{px^2+qx+r}\,dx$

  put $ax^2+bx+c=l(px^2+qx+r)+m\left(\frac{d}{dx}(px^2+qx+r)\right)+n$

  Find l, m and n by equating coefficients of like powers of x and then split the integral into three integrals.

- **Trigonometric integrals**

  $\int \frac{a\sin x+b\cos x}{c\sin x+d\cos x}\,dx$

  Put a sin$x$ + b cos$x$ = L (Denominator) + M (Derivative of denominator)

  Note : (1) To evaluate the integration of the forms

  $\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$, $\int \frac{dx}{a+b\sin^2 x}$, $\int \frac{dx}{a+b\cos^2 x}$, $\int \frac{dx}{(a\sin x+b\cos x)^2}$ and $\int \frac{dx}{a+b\sin^2 x+c\cos^2 x}$

  Step 1 : Divide by $\cos^2 x$ in each case.

  Step 2 : Put $\tan x = t$, we get the form $\int \frac{dx}{at^2+bt+c}$ for which the method to evaluate is already discussed.

  (2) To evaluate integrals of the form

  (i) $\int \frac{dx}{a\sin x+b\cos x}$ (ii) $\int \frac{dx}{a+b\sin x}$

  (iii) $\int \frac{dx}{a+b\cos x}$ (iv) $\int \frac{dx}{a\sin x+b\cos x+c}$

  For all the cases (i), (ii), (iii) & (iv), take universal substitution $\tan\frac{x}{2}=t$ and $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$, $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ are used. This substitution convert the integrals in the form $\int \frac{dt}{at^2+bt+c}$ and to evaluate such integral, method is already discussed.

- **Properties of Definite Integrals**

(i) $\int_{\alpha}^{\beta} f(x)\,dx = \int_{\alpha}^{\beta} f(t)\,dt$

(ii) $\int_{\alpha}^{\beta} f(x)\,dx = -\int_{\beta}^{\alpha} f(x)\,dx$

(iii) (a) $\int_{\alpha}^{\beta} f(x)\,dx = \int_{\alpha}^{\gamma} f(x)\,dx + \int_{\gamma}^{\beta} f(x)\,dx,\ \alpha < \gamma < \beta$

(b) $\int_{\alpha}^{\beta} f(x)\,dx = \int_{\alpha}^{c_1} f(x)\,dx + \int_{c_1}^{c_2} f(x)\,dx + \ldots + \int_{c_n}^{\beta} f(x)\,dx,$

$\alpha < c_1 < c_2 < c_3 \ldots\ldots < c_{n-1} < c_n < \beta$

(iv) $\int_{0}^{\alpha} f(x)\,dx = \int_{0}^{\alpha} f(\alpha - x)\,dx$

(v) $\int_{\alpha}^{\beta} f(x)\,dx = \int_{\alpha}^{\beta} f(\alpha + \beta - x)\,dx$

(vi) $\int_{-\alpha}^{\alpha} f(x)\,dx = \begin{cases} 2\int_{0}^{\alpha} f(x)dx, & \text{if } f(-x) = f(x) \\ 0, & \text{if } f(-x) = -f(x) \end{cases}$

(vii) $\int_{0}^{2a} f(x)\,dx = \begin{cases} 2\int_{0}^{\alpha} f(x)dx, & \text{if } f(2\alpha - x) = f(x) \\ 0, & \text{if } f(2\alpha - x) = -f(x) \end{cases}$

(viii) If $f(t)$ is an odd function then $g(x) = \int_{0}^{x} f(t)\,dt$ is an even function.

(ix) If $f(t)$ is an even function then $g(x) = \int_{0}^{x} f(t)\,dt$ is an odd function.

- **Definite integral as the limit of a sum**

An alternative method of finding $\int_{\alpha}^{\beta} f(x)\,dx$ is that the definite integral $\int_{\alpha}^{\beta} f(x)\,dx$ is a limiting case of the summation of an infinite series provided $f(x)$ is continuous on $[\alpha, \beta]$, *i.e.*,

$$\int_{\alpha}^{\beta} f(x)\,dx = \lim_{h \to 0} h[f(\alpha) + f(\alpha + h) + \ldots + f(\alpha + (n-1)h)]$$

where $h = \frac{\beta - \alpha}{n}$

- **Formation of differential equations whose general solution is given**

If an equation in independent and dependent variables involving some arbitrary constants is given, then a differential equation is obtained as follows :

(i) Differentiate the given equation w.r.t. the independent variable (say x)as many times as the number of arbitrary constants in it.

(ii) Eliminate the arbitrary constants.

(iii) The eliminant is the required differential equation.

- **Methods of solving the first order and first degree differential equation**

Type (1) : Differential equation of the form $\frac{dy}{dx} = f(x)$

To solve such type of equations we do the following : $\frac{dy}{dx} = f(x) \Leftrightarrow dy = f(x)\,dx$ ......(A)

Now integrating (A) both sides, we get

$y = \int f(x)dx + k$, k is some constant.

Type (2) : Equation in variable separable form.

Consider the equation $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ which can be written as $g(y)\,dy = f(x)\,dx$

Now integrating both sides we get the solution :

$\int g(y)dy = \int f(x)dx + k$, k is some constant.

Type (3) : Equation reducible to homogeneous form. Equation of the form $\frac{dy}{dx} = F(x, y)$ where $F(x, y)$ is homogeneous function of degree greater than zero, then we make substitution $y = vx$ and $\frac{dy}{dx} = v + \frac{xdv}{dx}$

Type (4) : Solution of linear differential equation

(a) Consider $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x or constant only.

we solve such equation by finding $e^{\int Pdx}$ where $e^{\int Pdx}$ is known as integrating factor.

$\therefore$ The solution is $y \cdot (\text{I.F.}) = \int (Q \times \text{I.F.})\,dx + c$

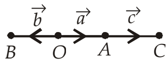
(b) Consider $\frac{dx}{dy} + Px = Q$, where P, Q are functions of y or constant only then $\text{I.F.} = e^{\int Pdy}$

$\therefore$ The solution is $x \cdot (\text{I.F.}) = \int (Q \times \text{I.F.})\,dy + c$

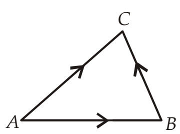
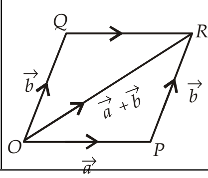
VECTOR ALGEBRA

- Vectors have magnitude and direction denoted by $\overrightarrow{AB}$ whereas scalars have only magnitude. The magnitude of vector is the length of the line segment AB denoted by $|\overrightarrow{AB}|$.

Types of Vectors

Types of Vectors	Definition	Notation
(i) Zero vector/ Null vector	Initial point and terminal point coincide	$\overrightarrow{AA}$
(ii) Unit vector	Magnitude is unity	$\hat{a}$
(iii) Coinitial vectors	Vectors having same initial point.	$\overrightarrow{OA}, \overrightarrow{OC}, \overrightarrow{OD}$
(iv) Collinear vectors	Vectors which are parallel to the same vector	
(v) Equal vectors	Vectors having same magnitude and same direction	$\vec{a} = \vec{b}$
(vi) Negative of a vector	Vector having same magnitude but opposite direction	$\overrightarrow{BA} = -\overrightarrow{AB}$
(vii)Free vectors	Vectors whose initial point is not specified	

Laws of Addition of Vectors

(i) Triangle Law of Addition		$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
(ii) Parallelogram Law of Addition		$\overrightarrow{OP} + \overrightarrow{OQ} = \overrightarrow{OR}$

Properties of Addition of Vectors

(i)	$\vec{a} + \vec{b} = \vec{b} + \vec{a}$	(Commutativity)
(ii)	$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$	(Associativity)
(iii)	$\vec{a} + \vec{0} = \vec{a}$	(Additive identity)
(iv)	$\vec{a} + (-\vec{a}) = \vec{0}$	(Additive inverse)

Multiplication of a Vector by a Scalar

- Let $\vec{a}$ be a vector and m be a scalar, then multiplication of vector $\vec{a}$ by scalar m is given by $(m\vec{a}) = (m)(\vec{a})$

Properties of Multiplication of Vectors by a Scalar

(i) $m(-\vec{a}) = -(m\vec{a})$ (ii) $(-m)(-\vec{a}) = m\vec{a}$

(iii) $m(n\vec{a}) = n(m)\vec{a} = (mn)\vec{a}$ (iv) $(m+n)\vec{a} = m\vec{a} + n\vec{a}$

(v) $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$

- Two vectors $\vec{a}$ and $\vec{b}$ are collinear or parallel iff $\vec{a} = m\vec{b}$ for some non zero scalar m.
- **Position Vector:** Position vector of a point $P(x, y, z)$ is given as $\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$ and its magnitude as $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$, where O is the origin.

Components of a Vector in Two Dimension

If a point P in a plane has coordinate (x, y) then

(i) $\overrightarrow{OP} = x\hat{i} + y\hat{j}$ (ii) $|\overrightarrow{OP}| = \sqrt{x^2 + y^2}$

(iii) The component of $\overrightarrow{OP}$ along x-axis is a vector $x\hat{i}$, whose magnitude is $|x|$ and whose direction is along OX or OX' according as x is positive or negative.

(iv) The component of $\overrightarrow{OP}$ along y-axis is a vector $y\hat{j}$, whose magnitude is $|y|$ and whose direction is along OY or OY' according as y is positive or negative.

For any two vectors, $\vec{a} = x_1\hat{i} + y_1\hat{j}$ and $\vec{b} = x_2\hat{i} + y_2\hat{j}$

(i) $\vec{a} + \vec{b} = (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j}$

(ii) $\vec{a} - \vec{b} = (x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j}$

(iii) $m\vec{a} = (mx_1)\hat{i} + (my_1)\hat{j}$, where m is a scalar quantity

(iv) $\vec{a} = \vec{b} \Leftrightarrow x_1 = x_2$ and $y_1 = y_2$

Components of a Vector in Three Dimensions

If $P(x, y, z)$ is a point in space and $\hat{i}, \hat{j}, \hat{k}$ are unit vectors, then

(i) $\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

(ii) $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$

(iii) The component vectors of $\overrightarrow{OP}$ along x, y and z are vectors $x\hat{i}, y\hat{j},$ and $z\hat{k}$.

- **Vector Joining Two Points :** If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two points, then vector joining P and Q is given by
$\overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$.

Section Formula

- The position vector of a point R dividing the line segment joining the points P and Q, whose position vectors are $\vec{a}$ and $\vec{b}$ respectively
  (i) in the ratio $m : n$ internally is $\frac{m\vec{b}+n\vec{a}}{m+n}$
  (ii) in the ratio $m : n$ externally is $\frac{m\vec{b}-n\vec{a}}{m-n}$

- Projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ and the projection vector of $\vec{b}$ on $\vec{a}$ is $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)$

- **Direction Cosines :** If α, β, γ are the angles made by a vector with the positive directions of x, y and z axes respectively, then $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called direction cosines of the vector and are generally denoted by the letters l, m, n respectively.
$\therefore\ l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$

- **Direction Ratios :** If l, m, n are the direction cosines of a vector and a, b, c are three numbers such that $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$. Then, direction ratios are proportional to a, b, c.

- If α, β, γ are direction angles of vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, then

$$\cos\alpha = \frac{a_1}{|\vec{a}|}, \cos\beta = \frac{a_2}{|\vec{a}|}, \cos\gamma = \frac{a_3}{|\vec{a}|}$$

Product of Two Vectors

- **Scalar (or Dot) Product of Two Vectors :** It is defined as $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$.

Properties of scalar product

(i) $\vec{a}\cdot\vec{b}$ is a real number.
(ii) $\vec{a}\cdot\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$
(iii) If $\theta = 0$, then $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$
(iv) If $\theta = \pi$, then $\vec{a}\cdot\vec{b} = -|\vec{a}||\vec{b}|$
(v) For mutually perpendicular unit vectors $\hat{i}, \hat{j}, \hat{k}$, we have
$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
(vi) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$
(vii) $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$
(viii) $(\lambda\vec{a})\cdot\vec{b} = \vec{a}\cdot(\lambda\vec{b})$

- **Vector (or Cross) Product of Two Vectors :** It is defined as $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, where θ is the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$.

Properties of vector product

(i) $\vec{a}\times\vec{b}$ is a vector
(ii) $\vec{a}\times\vec{b} = \vec{0} \Leftrightarrow \vec{a} \parallel \vec{b}$
(iii) When $\theta = 0$, $\vec{a}\times\vec{b} = \vec{0}$
(iv) When $\theta = \pi$, $\vec{a}\times\vec{b} = \vec{0}$
(v) When $\theta = \frac{\pi}{2}, \vec{a}\times\vec{b} = |\vec{a}||\vec{b}|$
(vi) For mutually perpendicular unit vectors
$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$; $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$
(vii) Area of triangle $ABC = \frac{1}{2}|\vec{a}\times\vec{b}|$
(viii) $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$
(ix) $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$ *i.e.*, $\vec{a}\times\vec{b} \ne \vec{b}\times\vec{a}$
(x) $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$
(xi) Area of parallelogram $ABCD = |\vec{a}\times\vec{b}|$

- If θ is angle between $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

THREE DIMENSIONAL GEOMETRY

- The direction cosines of the line are

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}},\ m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}},$$

$n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$

where a, b, c are direction ratios

Relation between the direction cosines of a line is $l^2 + m^2 + n^2 = 1$

Direction cosines of a line passing through two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$\frac{x_2-x_1}{PQ}, \frac{y_2-y_1}{PQ}, \frac{z_2-z_1}{PQ}$$

where $PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

- **Section formula :** Ratio is $m : n$

(i) Internal division :

$$x = \frac{mx_2+nx_1}{m+n}, y = \frac{my_2+ny_1}{m+n}, z = \frac{mz_2+mz_1}{m+n}$$

(ii) External division :

$$x = \frac{mx_2-nx_1}{m-n}, y = \frac{my_2-ny_1}{m-n}, z = \frac{mz_2-mz_1}{m-n}$$

(iii) Mid point formula :

$$x = \frac{x_1+x_2}{2}, y = \frac{y_1+y_2}{2}, z = \frac{z_1+z_2}{2}$$

- **Angle between two lines :**

(i) Vector form : $\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

(ii) Cartesian form :

$$\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

- Two lines are

(i) perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

(ii) parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Equation of a line passing through a given point and parallel to a given vector

(i) Vector Equation : $\vec{r} = \vec{a} + \lambda\vec{b}$

(ii) Cartesian Equation : $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

(iii) If l, m, n are the direction cosines of the line, the equation of the line is

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

Equation of a line passing through two given points

(i) Vector equation: $\vec{r} = \vec{a} + \lambda(\vec{b}-\vec{a}), \lambda \in R$

(ii) Cartesian equation:

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

- **Shortest distance between two lines :**

(a) Distance between two skew lines

(i) Vector form : $d = \left|\frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

(ii) Cartesian form :

$$\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right|$$

(b) Distance between parallel lines

$$d = \left|\frac{\vec{b} \times (\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$$

Plane

- **Equation of a plane in normal form**

(i) Vector form : $\vec{r}\cdot\hat{n} = d$

(ii) Cartesian form : $lx + my + nz = d$

- **Equation of a plane perpendicular to a given vector and passing through a given point.**

(i) Vector form : $\vec{r}\cdot\hat{n} = d$

(ii) Cartesian form :

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

- **Equation of a plane passing through three non collinear points :**

(i) Vector form : $(\vec{r}-\vec{a}).[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})] = 0$

(ii) Cartesian form :

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

- **Intercept form of the equation of a plane :**

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$$

- **Equation of the plane passing through the intersection of two given planes**

(i) Vector form : $\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2) = d_1 + \lambda d_2$

(ii) Cartesian form : $(a_1x + b_1y + c_1z - d_1) + \lambda(a_2x + b_2y + c_2z - d_2) = 0$

- **Coplanarity of two lines**

 (i) Vector form : $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 0$

 (ii) Cartesian form :

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Angle between two planes

 (i) Vector form : $\cos\theta = \left|\dfrac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|$

 (ii) Cartesian form :

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

- **Distance of a point from a plane**

 (i) Vector form : $= \dfrac{|\vec{a}\cdot\vec{N} - d|}{|\vec{N}|}$, where $\vec{N}$ is normal to the plane.

 (ii) Cartesian form : $\left|\dfrac{ax_1 + by_1 + cz_1 - d}{\sqrt{a^2 + b^2 + c^2}}\right|$

- Angle between a line and a plane

 Vector form : $\phi = \sin^{-1}\left|\dfrac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}\right|$

LINEAR PROGRAMMING

- **Graphical solution of L.P.P**
 This method of solving linear programming problem is referred as **Corner Point method.** This method comprises of following steps:

1. Find the feasible region of the L.P.P and determine its corner points (vertices).

2. Evaluate the objective function $Z = ax + by$ at each corner point. Let M and m respectively be the largest and smallest values at these points.

3. **(a)** If the feasible region is bounded, M and m respectively are the maximum and minimum values of the objective function.

 (b) If the feasible region is unbounded, then

 (i) M is the maximum value of the objective function, if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, the objective function has no maximum value.

 (ii) Similarly m is the minimum value of the objective function, if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, the objective function has no minimum value.

Note : If two corner points of the feasible region are both optimal solutions of the same type, *i.e.*, both produce the same maximum or minimum, then any point on the line segment joining these two points is also an optimal solution of the same type.

PROBABILITY

Terms	Definition
Conditional Probability	If E_1 and E_2 are any two events, then conditional probability of E_2 when Probability E_1 has already occured is given by $P(E_1 \mid E_2) = \dfrac{P(E_1 \cap E_2)}{P(E_2)}$, $P(E_2) \neq 0$ **Properties of Conditional Probability :** (i) The conditional probability of an event E given that F has already occurred lies between 0 and 1. (ii) Let E and F be events of a sample space S of an experiment, then $P(S \mid F)\ P(F \mid F) = 1$ (iii) If A and B are any two events of a sample space S and F is an event of S such that $P(F) \neq 0$, then $P((A \cup B) \mid F) = P(A \mid F) + P(B \mid F) - P((A \cap B) \mid F)$ In particular, if A and B are disjoint events, then $P((A \cup B) \mid F) = P(A \mid F) + P(B \mid F)$ (iv) $P(E' \mid F) = 1 - P(E \mid F)$
Multiplication Theorem on Probability	(i) For two events : If E_1 and E_2 are two events then $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2 \mid E_1) = P(E_2) \cdot P(E_1 \mid E_2)$ where $P(E_1) \neq 0$ and $P(E_2) \neq 0$ (ii) For n events $P(E_1 \cap E_2 \cap ... \cap E_n) = P(E_1) \cdot P(E_2/E_1) \cdot P(E_3 \mid E_1 \cap E_2) ... P(E_n \mid E_1 \cap E_2 \cap ... \cap E_{n-1})$
Independent Events	(i) Two events E_1 and E_2 are called independent if $P(E_1 \mid E_2) = P(E_1)$, $P(E_2) \neq 0$ and $P(E_2 \mid E_1) = P(E_2)$, $P(E_1) \neq 0$. (ii) For independent events E_1 and E_2, $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

Theorem of Total Probability	Let $\{E_1, E_2, ..., E_n\}$ be a partition of the sample space S, and suppose that each of the events $E_1, E_2, ..., E_n$ has non-zero probability of occurrence. Let A be any event associated with S, then $P(A) = P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2) + ... + P(E_n)\ P(A \mid E_n) = \sum_{i=1}^{n} P(E_i) \cdot P(A \mid E_i)$
Bayes' Theorem	If $E_1, E_2, ..., E_n$ are n non-empty events which constitute a partition of sample space S, *i.e.* $E_1, E_2, ..., E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup ... \cup E_n = S$ and A is any event of non-zero probability, then $P(E_i \mid A) = \dfrac{P(E_i)P(A \mid E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A \mid E_i)}$ for $i = 1, 2, 3, ..., n$
Random Variable	It is a function whose domain is a sample space and whose range is some set of real numbers. It is often denoted by X. **Types of Random Variable** (i) If random variable takes countable number of distinct values it is called discrete variable. (ii) If random variable takes an infinite number of possible values it is called continuous variable.
Probability Distribution of Random Variable	$\begin{array}{\|c\|c\|c\|c\|}\hline X & x_1 & x_2 & x_3..x_n \\ \hline P(x) & p_1 & p_2 & p_3..p_n \\ \hline\end{array}$, where $p_i > 0$, $\sum_{i=1}^{n} p_i = 1$, $i = 1, 2, 3, ..., n$ where $x_1, x_2, x_3, ..., x_n$ are possible values and respective probabilities $p_1, p_2, p_3, ..., p_n$ of random variable X.
Mean of Random Variable	The mean of a random variable X is also called expectation of X, denoted $E(X) = \mu = x_1p_1 + x_2p_2 + ... x_np_n = \sum_{i=1}^{n} x_ip_i$
Variance of a Random Variable	Let X be a random variable whose possible values $x_1, x_2, ..., x_n$ occur with probabilities $p(x_1), p(x_2), ..., p(x_n)$ respectively $\sigma^2_x = V(x) = \sum_{i=1}^{n} x_i^2 p_i - \left(\sum_{i=1}^{n} x_ip_i\right)^2$
Standard Deviation	$\sigma_x = \sqrt{V(x)}$
Bernoulli Trials	Trials of a random experiment are Bernoulli trials, if (i) They are finite. (ii) They are independent of each other. (iii) Each trial has two outcomes : success or failure. (iv) Probability of success or failure remains the same in each trial.
Binomial Distribution	A random variable X which takes values 0, 1, 2, ..., n follows binomial distribution if its probability distribution function is given by $P(X = r) = {}^nC_r\, p^r\, q^{n-r}$, $r = 0, 1, 2$ ..., n, where $p + q = 1$, $p, q > 0$, p = probability of success, q = probability of failure, n = number of trials.
Mean, Variance and Standard Deviation of Binomial Distribution	Mean = np, Variance = npq and Standard Deviation = $\sqrt{npq}$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