BITSAT
MOCK TEST PAPER

Exam Date 16th to 26th May

Time : 3 Hours *Max. Marks : 450*

SECTION-I (PHYSICS)

1. Dimensions of ohm are same as (where h is Planck's constant and e is charge)

(a) $\frac{h}{e}$ (b) $\frac{h^2}{e}$ (c) $\frac{h}{e^2}$ (d) $\frac{h^2}{e^2}$

2. A player throws a ball upwards with an initial speed of 30 m s^{-1}. To what height does the ball rise?
(Take g = 10 m s^{-2})

(a) 30 m (b) 45 m (c) 90 m (d) 100 m

3. A body of mass 0.5 kg travels in a straight line with velocity $v = kx^{3/2}$ where k = 5 m$^{-1/2}$ s^{-1}. What is the work done by the net force during its displacement from x = 0 to x = 2 m?

(a) 25 J (b) 50 J (c) 100 J (d) 150 J

4. There is some change in length when a 33000 N tensile force is applied on a steel rod of area of cross-section 10^{-3} m^2. The change in temperature required to produce the same elongation if the steel rod is heated is
(The modulus of elasticity is 3×10^{11} N m^{-2} and coefficient of linear expansion of steel is 1.1×10^{-5} °C^{-1})

(a) 20°C (b) 15°C (c) 10°C (d) 0°C

5. The function $\sin\omega t - \cos\omega t$ represents

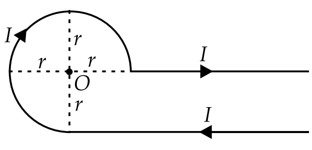
(a) a simple harmonic motion with a period $\frac{\pi}{\omega}$

(b) a simple harmonic motion with a period $\frac{2\pi}{\omega}$

(c) a periodic, but not simple harmonic motion with a period $\frac{\pi}{\omega}$

(d) a periodic, but not simple harmonic motion with a period $\frac{2\pi}{\omega}$

6. Current I is flowing in a conductor shaped as shown in the figure. The radius of the curved part is r and the length of straight portion is very large. The value of the magnetic field at the centre O will be



(a) $\frac{\mu_0 I}{4\pi r}\left(\frac{\pi}{2}+1\right)$

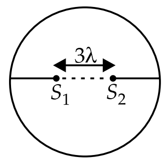
(b) $\frac{\mu_0 I}{4\pi r}\left(\frac{\pi}{2}-1\right)$

(c) $\frac{\mu_0 I}{4\pi r}\left(\frac{3\pi}{2}+1\right)$ (d) $\frac{\mu_0 I}{4\pi r}\left(\frac{3\pi}{2}-1\right)$

7. Two conducting spheres of radii 5 cm and 10 cm are given a charge of 15 μC each. After connecting the spheres by a copper wire, the charge on the smaller sphere is equal to

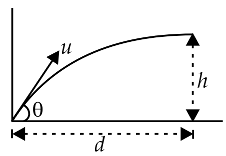
(a) 20 μC (b) 5 μC (c) 10 μC (d) 15 μC

8. If two coherent sources are placed at a distance 3λ from each other, symmetric to the centre of the circle of radius R as shown in the figure ($R >> \lambda$), then number of bright fringes shown on the screen placed along the circumference is



(a) 16
(b) 12
(c) 8
(d) 4

9. If a stone is to hit at a point which is at a distance d away and at a height h above the point from where the stone starts as shown in the figure then what is the value of initial speed u if stone is launched at an angle θ?



(a) $\frac{g}{\cos\theta}\sqrt{\frac{d}{2(d\tan\theta - h)}}$

(b) $\frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$

(c) $\sqrt{\frac{gd^2}{h\cos^2\theta}}$ (d) $\sqrt{\frac{gd^2}{(d-h)}}$

10. Magnification at least distance of distinct vision of a simple microscope having its focal length 5 cm is

(a) 2 (b) 4 (c) 5 (d) 6

11. A point charge causes an electric flux of -1.0×10^3 N m^2/C to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. If the radius of the Gaussian surface was doubled, how much flux would pass through the surface?

(a) -1.0×10^3 N m^2/C (b) -2.0×10^3 N m^2/C
(c) -3.0×10^3 N m^2/C (d) -4.0×10^3 N m^2/C

12. A circular coil of radius 10 cm, 500 turns and resistance 2 Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. The current induced in the coil is
(Horizontal component of the earth's magnetic field at the place is 3.0×10^{-5} T)

(a) 1.9×10^{-3} A (b) 2.9×10^{-3} A
(c) 3.9×10^{-3} A (d) 4.9×10^{-3} A

13. The escape velocity for a planet is v_e. A particle is projected from its surface with a speed v. For this particle to move as a satellite around the planet,
(a) $\frac{v_e}{2} < v < v_e$ (b) $\frac{v_e}{\sqrt{2}} < v < v_e$
(c) $v_e < v < \sqrt{2}v_e$ (d) $\frac{v_e}{\sqrt{2}} < v < \frac{v_e}{2}$

14. A and B are two points on the axis and the perpendicular bisector respectively of an electric dipole. A and B are far away from the dipole and at equal distances from it. The potentials at A and B are V_A and V_B respectively. Then,
(a) $V_A = V_B = 0$ (b) $V_A = 2V_B$
(c) $V_A \neq 0,\ V_B = 0$ (d) $V_A \neq 0,\ V_B \neq 0$

15. A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?
(a) 1.1×10^3 rad/s (b) 2.1×10^3 rad/s
(c) 3.1×10^3 rad/s (d) 4.1×10^3 rad/s

16. In a Young's double slit experiment, let S_1 and S_2 be the two slits, and C be the centre of the screen. If $\angle S_1CS_2 = \theta$ and λ is the wavelength, the fringe width will be
(a) $\frac{\lambda}{\theta}$ (b) $\lambda\theta$ (c) $\frac{2\lambda}{\theta}$ (d) $\frac{\lambda}{2\theta}$

17. A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?
(a) 1 s (b) 0.5 s (c) 1.5 s (d) 2.5 s

18. Two containers of equal volume contain the same gas at pressures P_1 and P_2 and absolute temperatures T_1 and T_2 respectively. On joining the vessels, the gas reaches a common pressure P and a common temperature T. The ratio $\frac{P}{T}$ is
(a) $\frac{P_1}{T_1} + \frac{P_2}{T_2}$ (b) $\frac{1}{2}\left[\frac{P_1}{T_1} + \frac{P_2}{T_2}\right]$
(c) $\frac{P_1T_2 + P_2T_1}{T_1 + T_2}$ (d) $\frac{P_1T_2 - P_2T_1}{T_1 - T_2}$

19. Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?
(a) 330 Hz (b) 318 Hz (c) 324 Hz (d) 321 Hz

20. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 70 cm, what is the emf of the second cell?
(a) 1.5 V (b) 1.25 V (c) 2.0 V (d) 2.5 V

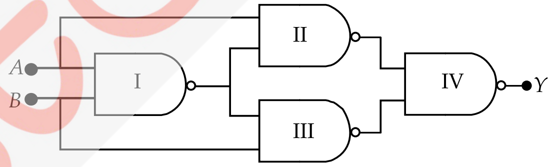
21. A prism of refractive index 1.5 is placed in water of refractive index 1.33. The refracting angle of a prism is 60°. What is the angle of minimum deviation in water? (Given sin 34° = 0.56)
(a) 4° (b) 8° (c) 12° (d) 16°

22. A particle is moving three times as fast as an electron. The ratio of the de Broglie wavelength of the particle to that of the electron is 1.813×10^{-4}. The mass of the particle is (Mass of electron = 9.1×10^{-31} kg)
(a) 1.67×10^{-27} kg (b) 1.67×10^{-31} kg
(c) 1.67×10^{-30} kg (d) 1.67×10^{-32} kg

23. The splitting of spectral lines under the effect of a magnetic field is called
(a) Zeeman effect (b) Bohr effect
(c) Heisenberg effect (d) magnetic effect

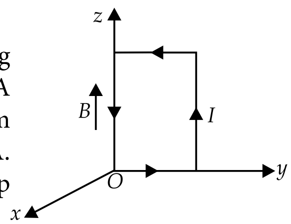
24. The ratio of the speed of the electron in the ground state of hydrogen atom to the speed of light in vacuum is
(a) $\frac{1}{2}$ (b) $\frac{2}{237}$ (c) $\frac{1}{137}$ (d) $\frac{1}{237}$

25. Select the output Y of the combination of gates shown in figure for inputs $A = 1, B = 0$; $A = 1, B = 1$ and $A = 0, B = 0$ respectively.



(a) (0, 1, 1) (b) (1, 0, 1)
(c) (1, 1, 1) (d) (1, 0, 0)

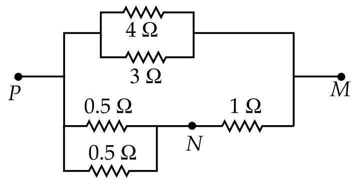
26. A uniform magnetic field of 3000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current 12 A. What is the torque on the loop as shown in the figure.



(a) 1.8×10^{-2} N m along negative y direction
(b) 1.8×10^{-2} N m along positive y direction
(c) 1.8×10^{-2} N m along positive z direction
(d) 1.8×10^{-2} N m along negative z direction

27. In the reaction, ${}^{24}_{12}\text{Mg} + {}^{4}_{2}\text{He} \rightarrow {}^{x}_{14}\text{Si} + {}^{1}_{0}n$, x is
(a) 28 (b) 27 (c) 26 (d) 22

28. In the circuit shown in figure the current through the 4 Ω resistor is 1 A when the points P and M are connected to a dc voltage source. The potential difference between points M and N is



(a) 0.5 V (b) 3.2 V (c) 1.5 V (d) 1.0 V

29. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. The focal length of the lens is

(a) $\frac{770}{36}$ cm (b) $\frac{77}{36}$ cm

(c) $\frac{36}{770}$ cm (d) $\frac{360}{77}$ cm

30. An inductance L, a capacitance C and a resistance R may be connected to an ac source of angular frequency ω, in three different combinations of RC, RL and LC in series. Assume that $\omega L = \frac{1}{\omega C}$. The power drawn by the three combinations are P_1, P_2, P_3 respectively. Then,

(a) $P_1 > P_2 > P_3$ (b) $P_1 = P_2 < P_3$

(c) $P_1 = P_2 > P_3$ (d) $P_1 = P_2 = P_3$

31. If the value of acceleration due to gravity at the surface of a sphere of radius r is a_m, then its value will be $a_m/3$ at distance ..... from centre

(a) $\sqrt{3}r$ (b) $\sqrt{3}/r$ (c) $2\sqrt{3}/r$ (d) $r/3$

32. A steel wire 1 m long and having 1 mm^2 cross section area is hung from a rigid support. When a weight of 1 kg is hung from it then change in length will be (Given $Y = 2 \times 10^{11}$ N m^{-2})

(a) 0.5 mm (b) 0.25 mm

(c) 0.05 mm (d) 5 mm

33. Agas is compressed adiabatically till its temperature is doubled. The ratio of its final volume to initial volume will be

(a) 1/2 (b) more than 1/2

(c) less than 1/2 (d) between 1 and 2

34. In the two vessels of same volume atomic hydrogen and helium at pressure 1 atm and 2 atm are filled. If temperature of both the samples is the same, then average speed of hydrogen atoms $< c_H >$ will be related to that of helium $< c_{He} >$ as

(a) $< c_H > = \sqrt{2} < c_{He} >$ (b) $< c_H > = < c_{He} >$

(c) $< c_H > = 2 < c_{He} >$ (d) $< c_H > = \frac{< c_{He} >}{2}$

35. A cylinder of radius r and thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius r and outer radius $2r$ whose thermal conductivity is K_2. There is no loss of heat across cylindrical surfaces when the ends of the combined system are maintained at temperatures T_1 and T_2. The effective thermal conductivity of the system, under steady state conditions is

(a) $\frac{(K_1K_2)}{(K_1+K_2)}$ (b) $(K_1 + K_2)$

(c) $\frac{(K_1+3K_2)}{4}$ (d) $\frac{(3K_1+K_2)}{4}$

36. A rod of length l and mass m is suspended in the middle by an inextensible string of length l and given torsional vibration. The time period of small oscillations is

(a) $\sqrt{\frac{l}{3g}}$ (b) $\sqrt{\frac{l}{12g}}$ (c) $\sqrt{\frac{m}{2g}}$ (d) $\sqrt{\frac{2l}{g}}$

37. A resonance tube is resonated with tuning fork of frequency 256 Hz. If the length of resonating air columns are 32 cm and 100 cm, then end correction will be

(a) 1 cm (b) 2 cm (c) 4 cm (d) 6 cm

38. $ABCD$ is a rectangle whose side AB = 10 cm and side BC = 24 cm. A charge of 0.104 μC is lying at the center O of rectangle. If the mid-point of side BC is E, then the work done in carrying 100 μC charge from B to E will be

(a) 1.152 J (b) 2.304 J (c) 4.082 J (d) 23.4 J

39. An achromatic combination of lenses has the power of convex lens 2 D and ratio of dispersive powers of lenses are equal then the focal length of concave lens is

(a) −50 cm (b) 100 cm (c) 20 cm (d) 200 cm

40. In an astronomical telescope in normal adjustment, a straight black line of length L is drawn on the objective lens. The eyepiece forms a real image of this line. The length of this image is l. The magnification of the telescope is

(a) $\frac{L}{l}$ (b) $\frac{L}{l}+1$ (c) $\frac{L}{l}-1$ (d) $\frac{L+1}{L-1}$

SECTION-II (CHEMISTRY)

41. Rutherford's experiment, which established the nuclear model of the atom, used a beam of

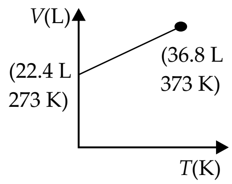
(a) β-particles, which impinged on a metal foil and got absorbed

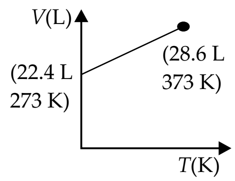
(b) γ-rays, which impinged on a metal foil and ejected electrons

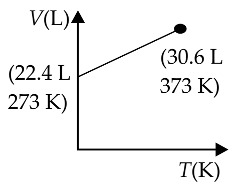
(c) helium atoms, which impinged on a metal foil and ejected electrons

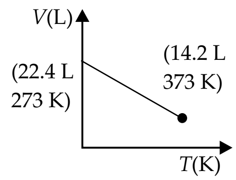
(d) helium nuclei, which impinged on a metal foil and got scattered.

42. Which of the following volume (V) - temperature (T) plots represents the behaviour of one mole of an ideal gas at one atmospheric pressure?

(a)


(b)


(c)


(d)


43. Which of the following will form a cell with the highest voltage?
(a) 1 M Ag^+, 1 M Co^{2+} (b) 2 M Ag^+, 2 M Co^{2+}
(c) 0.1 M Ag^+, 2 M Co^{2+} (d) 2 M Ag^+, 0.1 M Co^{2+}

44. Lemon juice normally has a pH of 2. If all the acid in the lemon juice is citric acid and there are no citrate salts present, then what will be the citric acid concentration (H.cit) in the lemon juice? (Assume that only the first hydrogen of citric acid is important).
(H.Cit $\rightleftharpoons$ H^+ + Cit^-. $K_a = 8.4 \times 10^{-4}$ mol lit^{-1})
(a) 8.4×10^{-4} M (b) 4.2×10^{-4} M
(c) 16.8×10^{-4} M (d) 12.0×10^{-2} M

45. One mole of an anhydrous salt *AB* dissolves in water with the evolution of 21.0 J mol^{-1} of heat. If the heat of hydration of *AB* is –29.4 J mol^{-1}, then the heat of dissociation of the hydrated salt *AB* is
(a) 50.4 J mol^{-1} (b) 8.4 J mol^{-1}
(c) – 50.4 J mol^{-1} (d) none of these.

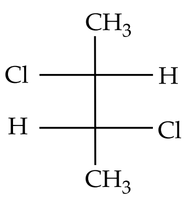
46. The solubility of A_2X_3 is y mol dm^{-3}. Its solubility product is
(a) $6\,y^4$ (b) $64\,y^4$ (c) $36\,y^5$ (d) $108\,y^5$

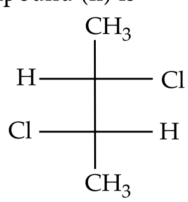
47. Acetophenone is prepared from
(a) Rosenmund reaction (b) Sandmeyer reaction
(c) Wurtz reaction (d) Friedel-Crafts reaction.

48. Which of the following compounds gives 2-iodopropane with HI?
(a) $CH_3CH_2CH_3$ (b) $CH_3CH{=}CH_2$
(c) $CH_2OHCHOHCH_2OH$ (d) C_2H_4

49. Which is the decreasing order of stability of the ions?
(i) $CH_3-\overset{+}{C}H-CH_3$ (ii) $CH_3-\overset{+}{C}H-OCH_3$
(iii) $CH_3-\overset{+}{C}H-COCH_3$
(a) (i) > (ii) > (iii) (b) (ii) > (iii) > (i)
(c) (iii) > (i) > (ii) (d) (ii) > (i) > (iii)

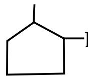
50. If optical rotation produced by the compound (i) is +52° then that produced by the compound (ii) is

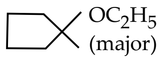
(i) 

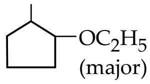
(ii) 

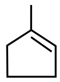
(a) –52° (b) +52°
(c) 0° (d) unpredictable.

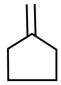
51. Potassium has a *bcc* structure with nearest neighbour distance 4.52 Å. Its atomic weight is 39. Its density will be
(a) 454 kg m^{-3} (b) 804 kg m^{-3}
(c) 852 kg m^{-3} (d) 910 kg m^{-3}

52. Br + $C_2H_5O^- \rightarrow Z$. *Z* may be

(a) 

(b) 

(c) 

(d) 

53. If urea is treated with thionyl chloride then we get
(a) $NH_2-\underset{Cl\quad Cl}{C}-NH_2$ (b) $NH_2-C\equiv N$
(c) $NH_2-\underset{Cl}{C}{=}NH$ (d) none of these.

54. How many double bond equivalents are possible for $C_7H_6O_2$?
(a) 5 (b) 6 (c) 7 (d) 8

55. Which one is called modified Cannizzaro reaction?
(a) MPV reaction (b) HVZ reaction
(c) Tischenko reaction (d) Birch reduction

56. dsp^2 hybridisation is in
(a) $[Ni(CN)_4]^{2-}$ (b) $[Ni(CO)_4]$
(c) $[Ni(NH_3)_4]$ (d) none of these.

57. Which one has maximum electrical conductance?
(a) $PtCl_4.4NH_3$ (b) $PtCl_4.5NH_3$
(c) $PtCl_4.6NH_3$ (d) $PtCl_4.3NH_3$

58. Radius of hydrated ion is in the order
(a) $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$
(b) $Li^+ < Na^+ < K^+ < Rb^+ < Cs^+$
(c) $Li^+ < Na^+ < K^+ < Rb^+ > Cs^+$
(d) $Li^+ < Na^+ > K^+ < Rb^+ > Cs^+$

59. If thio-cyanide ion is added to potash-ferric alum then red colour appears. This colour is due to the formation of
(a) KSCH (b) $Fe(SCN)_3$
(c) $Fe(SCN)_2$ (d) Fe(SCN)

60. Which of the following reactions is an example of disproportionation reaction?
(a) Tischenko reaction (b) Aldol condensation
(c) Rosenmund's reaction (d) Clemmensen reduction

61. Gold number is defined as
(a) amount of gold present in the colloid
(b) coagulating power of colloid
(c) amount of gold required to break the gold
(d) efficiency of the protective colloid.

62. A balloon filled with CO_2 has developed a small hole. It is quickly kept into a tank of hydrogen maintained at the same temperature. What happens?
(a) It will shrink (b) It will enlarge
(c) CO_2 reacts (d) No change

63. Lewis formula for the formate ion, HCO_2^- would show a total of X valence electrons with Y lone pairs on the carbon atom. X and Y correspond to
(a) 17, 0 (b) 17, 1
(c) 18, 0 (d) 18, 1

64. How many grams of urea (NH_2CONH_2) must be added to 1.0 kg of water to decrease the freezing point to – 6.0°C (K_f for water = 1.86°C kg mol^{-1})
(a) 11.16 g (b) 111.6 g (c) 193.5 g (d) 201.6 g

65. The method which does not result in sol destruction is
(a) electrophoresis
(b) addition of electrolyte
(c) diffusion through animal membrane
(d) mixing two oppositely charged sols.

66. A student accidently splashes few drops of conc. H_2SO_4 on his cotton shirt. After a while, the splashed parts blacken and the holes appear. This has happened because sulphuric acid
(a) dehydrates the cotton with burning
(b) causes the cotton to react with air
(c) heats up the cotton
(d) removes the elements of water from cotton.

67. Which of the following can give iodometric titration?
(a) Fe^{2+} (b) Cu^{2+}
(c) Pb^{2+} (d) Ag^{2+}

68. The elements which occupy the peaks of ionization enthalpy curve are
(a) Na, K, Rb, Cs (b) Na, Mg, Cl, I
(c) Cl, Br, I, F (d) He, Ne, Ar, Kr

69. The actinoids showing +7 oxidation state are
(a) U, Np (b) Pu, Am
(c) Np, Pu (d) Am, Cm

70. The hybridisation state of Fe in $[Fe(H_2O)_5NO]SO_4$ is (At. No. of Fe = 26)
(a) dsp^2 (b) sp^3d (c) sp^3d^2 (d) d^2sp^3

71. Which of the following alcohols has the lowest solubility in water?
(a) Methanol (b) Ethanol
(c) 1-Propanol (d) 1-Butanol

72. Which carbohydrate is used in silvering of mirrors?
(a) Fructose (b) Glucose
(c) Sucrose (d) Starch

73. Which of the following is not true?
(a) Ordinary water is electrolysed more rapidly than D_2O.
(b) D_2O freezes at lower temperature than H_2O.
(c) Reaction between H_2 and Cl_2 is much faster than D_2 and Cl_2.
(d) Bond dissociation energy for D_2 is greater than H_2.

74. Consider the following reaction,
$5H_2O_2 + xClO_2 + 2OH^- \rightarrow xCl^- + yO_2 + 6H_2O$
The reaction is balanced if
(a) $x = 5, y = 2$ (b) $x = 2, y = 5$
(c) $x = 4, y = 10$ (d) $x = 5, y = 5$

75. The atomic numbers of the metallic and non-metallic elements which are liquid at room temperature (298 K) respectively are
(a) 55,87 (b) 33, 87
(c) 35, 80 (d) 80, 35

76.

	Column I		Column II
A.	Milk	(p)	5.0
B.	Black coffee	(q)	6.8
C.	Human tears	(r)	2.2
D.	Lemon juice	(s)	7.4

(a) A-r; B-p; C-q; D-s (b) A-q; B-s; C-p; D-r
(c) A-q; B-p; C-s; D-r (d) A-r; B-p; C-s; D-q

77. The solubility of CO_2 in water increases with
(a) increase in temperature
(b) reduction of gas pressure
(c) increase in gas pressure
(d) increase in volume.

78. Mark the wrong statement about enzymes.
(a) Enzymes are biological catalysts.
(b) Each enzyme can catalyse a number of similar reaction.
(c) Enzymes are very efficient catalysts.
(d) Enzymes are needed only in very small amounts for their action.

79. Identify the state function among the following.
(a) q (b) $q - w$ (c) q/w (d) $q + w$

80. Which one of the following elements has zero valency?
(a) Platinum (b) Gold
(c) Sulphur (d) Neon

SECTION-III (ENGLISH AND LOGICAL REASONING)

Directions (Q. 81 to 83) : Read the passage carefully and choose the best answer to each question out of the four alternatives.

It is to progress in the human sciences that we must look to undo the evils which have resulted from a knowledge of the physical world hastily and superficially acquired by populations unconscious of the changes in themselves that the new knowledge has made imperative. The road to a happier world than any known in the past lies open before us if atavistic destructive passions can be kept in leash while the necessary adaptations are made. Fears are inevitable in our time, but hopes are equally rational and far more likely to bear good fruit. We must learn to think rather less of the dangers to be avoided than of the good that will lie within our grasp if we can believe in it and let it dominate our thoughts. Science, whatever unpleasant consequences it may have by the way, is in its very nature a liberator, a liberator of bondage to physical nature and is to come, a liberator from the weight of destructive passions. We are on the threshold of utter disaster or unprecedentedly glorious achievement. No previous age has been fraught with problems so momentous; and it is to science that we must look to for a happy future.

81. What does science liberate us from ?
It liberates us from ________ .
(a) idealistic hopes of a glorious future
(b) slavery to physical nature and from passions
(c) bondage to physical nature
(d) fears and destructive passions.

82. To carve out a bright future a man should ________ .
(a) cultivate a positive outlook
(b) analyse dangers that lie ahead
(c) try to avoid dangers
(d) overcome fears and dangers

83. If man's bestial yearning is controlled ________ .
(a) the future will be brighter than the present
(b) the future will be tolerant
(c) the present will be brighter than the future
(d) the present will become tolerant

Directions (Q. 84 and 85) : Choose one alternative which is opposite in meaning to the given word.

84. Flagitious
(a) Vapid (b) Innocent
(c) Frivolous (d) Ignorant

85. Ostentatious
(a) Ignorant (b) Unpretentious
(c) Awkward (d) Bankrupt

Directions (Q. 86 and 87) : In each of the following questions, choose from the given words below the two sentences, that word which has the same meaning and can be substituted for the words in bold in the two given sentences.

86. I. Sachin managed to **hold** the ball even though he had to run a long distance.
II. Anshul said that the plan was not as simple as I was making it out to be and there was some **trap** in it.
(a) Grab (b) Bring (c) Take (d) Catch

87. I. The message was written on a **piece** of paper.
II. Gautam got into a **fight** with his classmate.
(a) Scrap (b) Brush (c) Grab (d) Box

Directions (Q. 88 to 90) : In each of the following questions, find out which part/parts of the sentence has an error. If there is no mistake, the answer is 'No error'.

88. (A) In management, as you rise higher, / (B) the problems you face become more and more unstructured and you can't just fall back on / (C) the tools you had been / (D) taught. / (E) No error
(a) A and B (b) C only
(c) B only (d) E only

89. (A) If you are great at ideas but not very good at getting into / (B) the nitty gritty / (C) of things and implementing them, then you work on a team /(D) that has someone who can implement. / (E) No error
(a) A only (b) C only (c) A & C (d) E only

90. (A) While initial reports indicate that the brand has been / (B) well received at the capital / (C) it is still too early to say how much of an impact / (D) it will have in the long run. / (E) No error
(a) B only (b) C and D (c) D only (d) E only

Directions (Q. 91 to 95) : In each question below, some words are given, one of which may be wrongly spelt. Find out that word where the spelling is wrong.

91. (a) Thrift (b) Sbutle
(c) Slight (d) Shoot

92. (a) Bearable (b) Beautifull
(c) Beetle (d) Beautician

93. (a) Accommodation (b) Allergy
(c) Anxiety (d) Ankel

94. (a) Ludicruous (b) Logical
(c) Lonesome (d) Lovely

95. (a) Dainty (b) Damage
(c) Dairy (d) Dafodil

Directions : (Q. 96 and 97) : Read the following information carefully to answer the given questions :
Six films – P, Q, R, S, T and U are to be released on consecutive Fridays. The schedule of the release is to be in accordance with the following conditions :
(i) P must be released a week before T.
(ii) R must not be released immediately after the first release.
(iii) Q must be released on the Friday following the Friday on which U is released.
(iv) S must be released on fifth Friday and should not be immediately preceded by Q.
(v) T must not be released in the last.

96. Which of the following films preceded T ?
(a) P (b) Q (c) S (d) U

97. Which of the following films released immediately after Q ?
(a) P (b) R (c) T (d) U

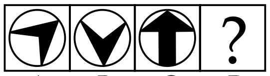
98. In a certain code, PLEADING is written as FMHCQMFB. How is SHOULDER written in the code ?
(a) KCDQTIPV (b) QDCKVPIT
(c) QDCKTIPV (d) TIPVQDCK

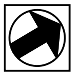
99. In a certain code, RAIL is written as KCTN and SPEAK is written as CGRUM. How will AVOID be written in that code ?
(a) FKQXC (b) KQXCF
(c) KRXCF (d) KQVCB

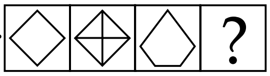
100. If '<' means 'minus', '>' means 'plus', '=' means 'multiplied by' and '\$' means 'divided by', then what would be the value of 27 > 81 \$ 9 < 6 ?
(a) 6 (b) 33 (c) 36 (d) 30

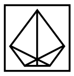
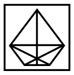
101. If '+' means 'divided by', '_' means 'added to', 'x' means 'subtracted from' and '÷' means 'multiplied by', then what is the value of 24 ÷ 12 – 18 + 9 ?
(a) – 25 (b) 0.72 (c) 15.30 (d) 290

Directions (Q. 102 and 103) : Figure A and B are related in a particular manner. Establish the same relationship between figure C and D by choosing a figure from amongst the four alternatives, which would replace the question mark in figure (D).

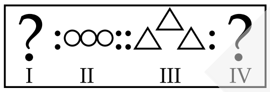
102. 
A B C D

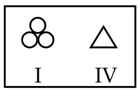
(a)  (b)  (c)  (d) 

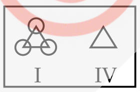
103. 
A B C D

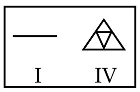
(a)  (b)  (c)  (d) 

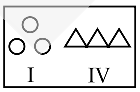
104. In the given question, there are four figures marked I, II, III and IV followed by four other figures (a), (b), (c) and (d), that each consists of two figures marked I and IV. Select a figure from given options, such that figure III is related to figure IV in the same way as figure I related to figure II.



(a) 

(b) 

(c) 

(d) 

105. Choose the odd numeral pair/group in the following question.
(a) 15 : 46 (b) 12 : 37 (c) 9 : 28 (d) 8 : 33

SECTION-IV (MATHEMATICS)

106. If z satisfies $|z+1| < |z-2|$, then $\omega = 3z + 2 + i$ satisfies
(a) $\omega + \bar{\omega} < 7$ (b) $|\omega + 1 + i| \le |\omega - 8 + i|$
(c) $\mathrm{Re}\left(\dfrac{1}{2\omega - 7}\right) > 0$ (d) $|\omega + 5| < |\omega - 4|$

107. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be
(a) an ellipse (b) a hyperbola
(c) a circle (d) none of these

108. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. The equation whose roots are α^{19}, β^7 is
(a) $x^2 - x - 1 = 0$ (b) $x^2 - x + 1 = 0$
(c) $x^2 + x - 1 = 0$ (d) $x^2 + x + 1 = 0$

109. The roots of the equation $(3 - x)^4 + (2 - x)^4 = (5 - 2x)^4$ are
(a) all real (b) all imaginary
(c) two real and two imaginary (d) none of these.

110. If $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$, then $a_1 + a_3 + a_5 + ... + a_{37}$ equals
(a) $2^{19}(2^{20} - 21)$ (b) $2^{20}(2^{19} - 19)$
(c) $2^{19}(2^{20} + 21)$ (d) none of these

111. If $x \in R$ and $n \in I$, then the determinant

$$\Delta = \begin{vmatrix} \sin(n\pi) & \sin x - \cos x & \log\tan x \\ \cos x - \sin x & \cos[(2n+1)\pi/2] & \log\cot x \\ \log\cot x & \log\tan x & \tan(n\pi) \end{vmatrix}$$

equal
(a) 0 (b) $\log\tan x - \log\cot x$
(c) $\tan(\pi/4 - x)$ (d) none of these

112. If $[y]$ denote the greatest integer $\le y$, and $2\left[\frac{x}{8}\right]^2 + 3\left[\frac{x}{8}\right] = 20$, then x lies in the smallest interval $[a, b)$ where $b - a$ is equal to
(a) 6 (b) 5
(c) 4 (d) none of these

113. The sum $\displaystyle\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$ (where $\binom{p}{q} = 0$ if $p < q$) is maximum where m is
(a) 5 (b) 10 (c) 15 (d) 20

114. If exp $\{(\tan^2 x - \tan^4 x + \tan^6 x - \tan^8 x + ...) \log_e 16\}$, $0 < x < \pi/4$, satisfies the quadratic equation $x^2 - 3x + 2 = 0$, then value of $\cos^2 x + \cos^4 x$ is
(a) 4/5 (b) 21/16 (c) 17/11 (d) 19/31

115. $\sin^2 \alpha + \cos^2 (\alpha + \beta) + 2 \sin\alpha \sin \beta \cos (\alpha + \beta)$ is independent of
(a) α (b) β
(c) both α and β (d) none of these

116. The function $f : R \to R$ defined by $f(x) = (x - 1)(x - 2)(x - 3)$ is
(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

117. The value of $n \in I$ for which the function $f(x) = \dfrac{\sin nx}{\sin(x/n)}$ has 4π as its period is
(a) 2 (b) 3
(c) 4 (d) 5

118. If $f(x) = \sin^2 x + \sin^2 (x + \pi/3) + \cos x \cos (x + \pi/3)$ and $g(5/4) = 1$, then $(gof)(x)$ is equal to
(a) 1 (b) 0
(c) $\sin x$ (d) none of these

119. Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then
(a) $a^2 + b^2 = p^2 + q^2$ (b) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$
(c) $a^2 + p^2 = b^2 + q^2$ (d) $\frac{1}{a^2}+\frac{1}{p^2}=\frac{1}{b^2}+\frac{1}{q^2}$

120. The curve described parametrically by $x = t^2 + t + 1$, $y = t^2 - t + 1$ represents
(a) a pair of straight lines (b) an ellipse
(c) a parabola (d) a hyperbola.

121. Locus of the midpoints of the chord of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ so that chord is always touching the circle $x^2 + y^2 = c^2$, $(c < a, c < b)$, is
(a) $(b^2 x^2 + a^2 y^2)^2 = c^2(b^4 x^2 + a^4 y^2)$
(b) $(a^2 x^2 + b^2 y^2)^2 = c^2(a^4 x^2 + b^4 y^2)$
(c) $(b^2 x^2 + a^2 y^2)^2 = c^2(b^2 x^2 + a^2 y^2)$
(d) none of these.

122. If polar of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is always touching the circle $x^2 + y^2 = c^2$, then locus of pole is
(a) $c^2 (a^4 y^2 + b^4 x^2) = a^4 b^4$
(b) $c^2 (a^4 x^2 + b^4 y^2) = a^4 b^4$
(c) $c^2 (a^4 y^2 + b^4 x^2 - a^3 b^3) = a^4 b^4$
(d) none of these.

123. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = 3\pi/2$, then the value of $x^{100}+y^{100}+z^{100}-\frac{3}{x^{101}+y^{101}+z^{101}}$ is
(a) 0 (b) 1 (c) 2 (d) 3

124. $\tan^{-1}\left(\frac{c_1x-y}{c_1y+x}\right)+\tan^{-1}\left(\frac{c_2-c_1}{1+c_2c_1}\right)+\tan^{-1}\left(\frac{c_3-c_2}{1+c_3c_2}\right)$ $+ .... + \tan^{-1}\left(\frac{1}{c_n}\right) = ...$
(a) $\tan^{-1}\left(\frac{x}{y}\right)$ (b) $\tan^{-1}\left(\frac{y}{x}\right)$
(c) $\cos^{-1}\left(\frac{x}{y}\right)$ (d) none of these

125. m integers are chosen at random with replacement from 1, 2, 3, ..., n. The probability that the largest chosen integer is k, is
(a) $\left(\frac{k}{n}\right)^m$ (b) $\left(\frac{k-1}{n}\right)^m$
(c) $\frac{1}{n}\left(\frac{k-1}{n}\right)^{m-1}$ (d) none of these.

126. Three persons A, B and C are to speak at a function along with 7 other persons. If the persons speak in a random order, the probability that A speaks before B, and B speaks before C, is
(a) $\frac{7!-3!}{10!}$ (b) $\frac{^{10}C_3}{10!}$
(c) $\frac{3!}{10!}$ (d) $\frac{1}{6}$

127. Let a, b, c be positive real numbers. Consider the equations
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,$
$\frac{-x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
The above system of equations has
(a) no solution
(b) a unique solution
(c) infinitely many solutions
(d) finitely many solutions

128. Let $f(x) = (x^2 - 1)^n g(x)$, $g(1) \neq 0$. Then, $f(x)$ has extrema at $x = 1$ if
(a) $n = 1$ (b) $n = 3$
(c) $n = 4$ (d) $n = 5$

129. The least value of the function $f(x) = 2 \log x - \log_x 0.01$, $x > 1$, is
(a) 1 (b) 2
(c) 3 (d) 4

130. Let $f(x)=\begin{cases} x^2-2x-2, & 0\le x<3 \\ 3-x, & 3\le x\le 5\end{cases}$. Then, we have
(a) least value = –3 (b) greatest value = 1
(c) minima at $x = 0$ (d) maxima at $x = 3$

131. $\int \frac{dx}{\sqrt{x}-\sqrt[3]{x}}$, is equal to
(a) $2x^{1/2} + 3x^{1/3} + x^{1/6} + \ln x + C$
(b) $2x^{1/2} + 3x^{1/3} + 6x^{1/6} + 6\ln x + C$
(c) $2x^{1/2} + 3x^{1/3} + 6\, x^{1/6} + 6\ln |x^{1/6} - 1| + C$
(d) none of these.

132. $\int 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x\, dx$, is equal to
(a) $2^{2^{2^x}} \cdot (\ln 2)^3 + C$ (b) $2^{2^x} \cdot (\ln 2)^2 + C$
(c) $\frac{2^{2^{2^x}}}{(\ln 2)^3} + C$ (d) none of these.

133. If $f(x)=\frac{1}{x^2}\int_2^x [t^2+f'(t)]dt = f(x)$, then $f'(2)$ equals
(a) –5/4 (b) 3
(c) 16/9 (d) 4/3

134. The left hand derivative of $f(x) = [x] \sin \pi x$ at $x = k$, $k \in I$, is
(a) $(-1)^k (k - 1)\pi$ (b) $(-1)^{k-1} (k - 1)\pi$
(c) $(-1)^k k\pi$ (d) $(-1)^{k-1} k\pi$

135. If $y=\cot^{-1}\left(\frac{\ln(e/x^2)}{\ln(ex^2)}\right)+\cot^{-1}\left(\frac{\ln(ex^4)}{\ln(e^2/x^2)}\right)$, then $\frac{d^2y}{dx^2}$ equals
(a) -1 (b) 0 (c) 1 (d) 2

136. If $\int_a^x f(t)dt-\int_0^y g(t)dt=b$, then the value of $\frac{dy}{dx}$ at (x_0, y_0) is
(a) $\frac{f(x_0)}{g(y_0)}$ (b) $\frac{g(x_0)}{f(y_0)}$
(c) $g(x_0)-f(y_0)$ (d) $f(x_0)-g(y_0)$

137. If the function $f(x)=\frac{\ln(x+a)}{\ln(x+b)}$, $(a,b>0)$ is strictly increasing, then
(a) $a>b$ (b) $b>a$
(c) $ab=1$ (d) $a=b$

138. If $f'(x)>0$ and $g'(x)<0\ \forall\ x\in R$, then which of the following does not hold?
(a) $fog\,(x)>fog\,(x+1)$ (b) $fog\,(x)>fog\,(x-1)$
(c) $fog(x)>gof(x+1)$ (d) $gof\,(x)>gof\,(x-1)$

139. The solution of differential equation
$yy'=x\left(\frac{y^2}{x^2}+\frac{\phi(y^2/x^2)}{\phi'(y^2/x^2)}\right)$ is
(a) $\phi\left(\frac{y^2}{x^2}\right)=cx^2$ (b) $x^2\,\phi\left(\frac{y^2}{x^2}\right)=c^2y^2$
(c) $x^2\,\phi\left(\frac{y^2}{x^2}\right)=c$ (d) $\phi\left(\frac{y^2}{x^2}\right)=\frac{cy}{x}$

140. If $xy=m^2-9$ be a rectangular hyperbola whose branches lie only in the second and fourth quadrant, then
(a) $|m|\geq 3$ (b) $|m|<3$
(c) $m\in R-\{|m|\}$ (d) none of these

141. The length of the latus-rectum of the parabola $ay^2+by=x-c$ is
(a) $1/a$ (b) $a/2$ (c) $1/4a$ (d) $4a$

142. The second degree equation $x^2+4y^2+2x+16y+13=0$ represent itself as
(a) a parabola (b) a pair of straight lines
(c) an ellipse (d) a hyperbola

143. Let $\vec{a}=\alpha_1\hat{i}+\alpha_2\hat{j}+\alpha_3\hat{k}$, $\vec{b}=\beta_1\hat{i}+\beta_2\hat{j}+\beta_3\hat{k}$, $\vec{c}=\gamma_1\hat{i}+\gamma_2\hat{j}+\gamma_3\hat{k}$ and $|\vec{a}|=2\sqrt{2}$ makes the angle $\frac{\pi}{3}$ with the plane of $\vec{b}$ and $\vec{c}$ and angle between is $\frac{\pi}{6}$, then the value of $\begin{vmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{vmatrix}^n$ equals
(a) $\left(\frac{|a||b|}{2\times 3}\right)^n$ (b) $\left(\frac{|\vec{c}||\vec{b}|}{\sqrt{3}\times 2}\right)^{n/2}$
(c) $\left(\frac{\sqrt{3}|\vec{b}||\vec{c}|}{\sqrt{2}}\right)^n$ (d) none of these

144. $(\vec{r}\cdot\hat{i})(\vec{r}\times\hat{i})+(\vec{r}\cdot\hat{j})(\vec{r}\times\hat{j})+(\vec{r}\cdot\hat{k})(\vec{r}\times\hat{k})$ is equal to
(a) $2\vec{r}$ (b) $\vec{r}$
(c) $4\vec{r}$ (d) $\vec{0}$

145. If $\sin x+\cos x+\tan x+\cot x+\sec x+\text{cosec}\,x=7$ and $\sin 2x=m-n\sqrt{7}$, then ordered pair (m, n) can be
(a) (6, 2) (b) (8, 3)
(c) (22, 8) (d) (11, 4)

146. If $x+y=\pi+z$, then $\sin^2x+\sin^2y-\sin^2z$ is equal to
(a) $2\sin x\sin y\sin z$ (b) $2\cos x\cos y\cos z$
(c) $2\sin x\cos y\cos z$ (d) $2\sin x\sin y\cos z$

147. Let R be a relation in N defined by $R=\{(x, y) : x+2y=8\}$, then range of R is
(a) {2, 4, 6} (b) {1, 2, 3, 4, 6}
(c) {1, 2, 3} (d) none of these

148. If R be a relation '<' from $A=\{1, 2, 3, 4\}$ to the set $B=\{1, 3, 5\}$ *i.e.* $(a, b)\in R\Leftrightarrow a<b$, then RoR^{-1} equals
(a) {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)}
(b) {(3, 3), (5, 3), (3, 5), (5, 5)}
(c) {(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)}
(d) {(4, 5), (3, 4), (3, 3)}

149. The straight lines joining the origin to the points of intersection of the straight line $hx+ky=2hk$ and the curve $(x-k)^2+(y-h)^2=c^2$ are at right angles, then
(a) $h^2+k^2+c^2=0$ (b) $h^2-k^2-c^2=0$
(c) $h^2+k^2-c^2=0$ (d) none of these

150. The exhaustive range of values of a such that the angle between the pair of tangents drawn from (a, a) to the circle $x^2+y^2-2x-2y-6=0$ lies in the range $(\pi/3, \pi)$ is
(a) $(1, \infty)$ (b) $(-5, -3)\cup(3, 5)$
(c) $(-\infty, -2\sqrt{2})\cup(2\sqrt{2}, \infty)$ (d) $(-3, -1)\cup(3, 5)$

SOLUTIONS

1. **(c) :** $\frac{h}{e^2} = \frac{[\mathrm{ML^2T^{-1}}]}{[\mathrm{AT}]^2} = [\mathrm{ML^2T^{-3}A^{-2}}]$

Resistance $= \frac{\text{Potential difference}}{\text{Current}} = \frac{[\mathrm{ML^2T^{-3}A^{-1}}]}{[\mathrm{A}]}$

$= [\mathrm{ML^2T^{-3}A^{-2}}]$

The SI unit of resistance is ohm.

Therefore, the dimensions of ohm are same as that of $\frac{h}{e^2}$.

2. **(b) :** Here, $u = 30$ m s^{-1}, $g = 10$ m s^{-2}

Using $v^2 - u^2 = 2as$

At maximum height, final velocity is zero *i.e.*, $v = 0$

$0 - (30)^2 = -2 \times 10 \times h$ or $h = \frac{30 \times 30}{20} = 45$ m

3. **(b) :** Given : $v = kx^{3/2}$

Acceleration, $a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx}$

As $v^2 = k^2x^3$

$\therefore\ 2v\frac{dv}{dx} = 3k^2x^2 \qquad \therefore\ a = \frac{3}{2}k^2x^2$

Force, $F = ma = \frac{3}{2}mk^2x^2$

Work done, $W = \int F dx = \int_0^2 \frac{3}{2}mk^2x^2 dx = \frac{3}{2}mk^2\left[\frac{x^3}{3}\right]_0^2$

$= \frac{3}{2} \times 0.5 \times 5^2 \times \frac{8}{3} = 50$ J

4. **(c) :** Young's modulus $Y = \frac{(F/A)}{\Delta l / l}$

or $\Delta l = \frac{(F/A)l}{Y}$ ...(i)

Also, $\Delta l = \alpha l \Delta\theta$ ...(ii)

As per question $\frac{(F/A)l}{Y} = \alpha l \Delta\theta$ or $\Delta\theta = \frac{F}{YA\alpha}$

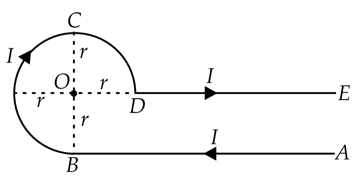
$= \frac{33000 \text{ N}}{(3 \times 10^{11} \text{ Nm}^{-2}) \times (10^{-3} \text{ m}^2) \times (1.1 \times 10^{-5}\ {}^\circ\text{C}^{-1})} = 10^\circ\text{C}$

5. **(b) :** $\sin\omega t - \cos\omega t = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right)$

$= \sqrt{2}\left(\sin\omega t \cos\frac{\pi}{4} - \cos\omega t \sin\frac{\pi}{4}\right) = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$

It represents SHM with a period $\frac{2\pi}{\omega}$.

6. **(c) :** For the circular part *BCD*, the angle subtended at the centre *O* is $3\pi/2$.



Total magnetic field at *O* is

$= B_{AB} + B_{BCD} + B_{DE}$

$= \frac{\mu_0}{4\pi}\frac{I}{r}[\sin 90^\circ + \sin 0^\circ] + \frac{\mu_0}{4\pi}\frac{I}{r} \times \frac{3\pi}{2} + 0$

$= \frac{\mu_0}{4\pi}\frac{I}{r} + \frac{\mu_0}{4\pi}\frac{I}{r} \times \frac{3\pi}{2} = \frac{\mu_0}{4\pi}\frac{I}{r}\left(1 + \frac{3\pi}{2}\right)$

7. **(c) :** Common potential $V = \frac{Q_1 + Q_2}{C_1 + C_2}$

$V = \frac{Q_1 + Q_2}{4\pi\varepsilon_0 r_1 + 4\pi\varepsilon_0 r_2} = \frac{Q_1 + Q_2}{4\pi\varepsilon_0 (r_1 + r_2)}$

$= \frac{30\ \mu\text{C}}{4\pi\varepsilon_0 (5 + 10) \times 10^{-2}} = \frac{30 \times 10^{-6} \times 9 \times 10^9}{15 \times 10^{-2}}$

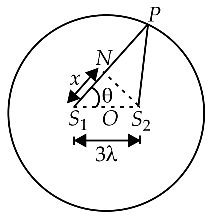
$V = 18 \times 10^5$ V

Charge on smaller sphere, $Q'_1 = 4\pi\varepsilon_0 r_1 V$

$= \frac{1}{9 \times 10^9} \times 5 \times 10^{-2} \times 18 \times 10^5$

$= 10 \times 10^3 \times 10^{-9} = 10 \times 10^{-6}$ C $= 10\ \mu$C

8. **(b) :** As is clear from figure, path difference between light waves from S_1 and S_2 is



$x = S_1N = S_1S_2\cos\theta = 3\lambda\cos\theta$

For constructive interference

$x = 3\lambda\cos\theta = n\lambda$

$\cos\theta = \frac{n}{3}$

Putting $n = 0, 1, 2, 3$

$\theta = 90^\circ, \theta = \cos^{-1}\frac{1}{3}, \theta = \cos^{-1}\frac{2}{3}; \theta = 0^\circ$

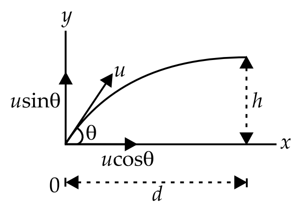
These are the positions of one quadrant. They repeat in all the four quadrants. Total number of distinct bright fringes = 12.

9. **(b) :** Motion along horizontal direction

$d = u\cos\theta \times t$

or $t = d/(u\cos\theta)$ ...(i)

Motion along vertical direction



$h = (u\sin\theta)t - \frac{1}{2}gt^2$ ...(ii)

Substitute the value of t in equation (ii), we get

$h = u\sin\theta \times \frac{d}{u\cos\theta} - \frac{1}{2}g\frac{d^2}{u^2\cos^2\theta}$

or $h = d\tan\theta - \frac{1}{2}g\frac{d^2}{u^2\cos^2\theta}$

or $\frac{1}{2}\frac{gd^2}{u^2\cos^2\theta} = d\tan\theta - h$

or $u^2 = \frac{gd^2}{2(d\tan\theta - h)\cos^2\theta}$

or $u = \frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$

10. **(d):** For final image at least distance of distinct visional

Magnification $M = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 6$

11. **(a) :** Here, $\phi_E = -1.0 \times 10^3$ N m^2/C, $r = 10.0$ cm

If the radius of Gaussian surface was doubled, flux passing through the surface would remain the same. This is because flux depends only on charge present inside the surface.

12. **(a) :** Initial magnetic flux through the coil,

$\phi_i = BA\cos\theta = 3.0 \times 10^{-5} \times (\pi \times 10^{-2}) \times \cos 0°$

$= 3\pi \times 10^{-7}$ Wb

Final magnetic flux after the rotation

$\phi_f = 3.0 \times 10^{-5} \times (\pi \times 10^{-2}) \times \cos 180° = -3\pi \times 10^{-7}$ Wb

Induced emf, $\varepsilon = -N\frac{d\phi}{dt} = -\frac{N(\phi_f - \phi_i)}{t}$

$$= -\frac{500\,(-3\pi \times 10^{-7} - 3\pi \times 10^{-7})}{0.25}$$

$$= \frac{500 \times (6\pi \times 10^{-7})}{0.25} = 3.8 \times 10^{-3} \text{ V}$$

$$I = \frac{\varepsilon}{R} = \frac{3.8 \times 10^{-3} \text{ V}}{2\,\Omega} = 1.9 \times 10^{-3} \text{ A}$$

13. **(b) :** For a satellite orbiting very close to the earth's surface, the orbital velocity $= \sqrt{Rg}$

This is equal to the velocity of projection and is the minimum velocity required to go into orbit. Also, the satellite would escape completely and not go into orbit for $v \geq v_e$.

where $v_e = \sqrt{2gR} = \sqrt{2}v$

$\therefore \quad \frac{v_e}{\sqrt{2}} < v < v_e$

14. **(c)**

15. **(a) :** Here, $C = 30$ μF $= 30 \times 10^{-6}$ F

$L = 27$ mH $= 27 \times 10^{-3}$ H

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}} = \frac{10^4}{9}$$

$= 1.1 \times 10^3$ rad/s

16. **(a) :** Fringe width, $\beta = \frac{\lambda D}{d}$ and $\theta = \frac{d}{D}$

$\therefore \quad \beta = \frac{\lambda}{\theta}$

17. **(b) :** Here, $M = 2.50$ kg, $T = 200$ N, $l = 20.0$ m

Mass per unit length, $\mu = \frac{M}{l} = \frac{2.5 \text{ kg}}{20.0 \text{ m}} = 0.125$ kg m^{-1}

Velocity, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{0.125}} = 40$ m s^{-1}

Time taken by disturbance to reach the other end,

$$t = \frac{l}{v} = \frac{20 \text{ m}}{40 \text{ m s}^{-1}} = 0.5 \text{ s}$$

18. **(b) :** For a closed system, the total mass of gas or the number of moles remains constant.

Let n_1 and n_2 be number of moles of gas in container 1 and container 2 respectively.

$P_1 V = n_1 R T_1$

or $\quad n_1 = \frac{P_1 V}{R T_1}$ ...(i)

$P_2 V = n_2 R T_2$

or $\quad n_2 = \frac{P_2 V}{R T_2}$ ...(ii)

$P(2V) = (n_1 + n_2)RT$ ...(iii)

Substituting the values of n_1 and n_2 in equation (iii), we get

$$P(2V) = \left(\frac{P_1 V}{R T_1} + \frac{P_2 V}{R T_2}\right) RT \text{ or } \frac{P}{T} = \frac{1}{2}\left(\frac{P_1}{T_1} + \frac{P_2}{T_2}\right)$$

19. **(b) :** Let original frequency of sitar string A be υ_A and original frequency of sitar string B be υ_B.

Number of beats per second = 6

$\therefore \quad \upsilon_B = \upsilon_A \pm 6 = 324 \pm 6 = 330$ or 318 Hz

When tension in A is reduced, its frequency reduces

$(\because \ \upsilon \propto \sqrt{T})$

Number of beats per second reduces to 3.

Therefore, frequency of $B = 324 - 6 = 318$ Hz.

20. **(d) :** Here, $\varepsilon_1 = 1.25$ V, $l_1 = 35.0$ cm

$\varepsilon_2 = ?$, $l_2 = 70$ cm

As $\frac{\varepsilon_2}{\varepsilon_1} = \frac{l_2}{l_1}$ or $\varepsilon_2 = \frac{\varepsilon_1 \times l_2}{l_1} = \frac{1.25 \times 70}{35} = 2.5$ V

21. **(b) :** Here, ${}^a\mu_g = 1.5 = \frac{3}{2}$, ${}^a\mu_w = 1.33 = \frac{4}{3}$

$A = 60°$

As ${}^a\mu_w \times {}^w\mu_g = {}^a\mu_g$ or ${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{(3/2)}{(4/3)} = \frac{9}{8}$

$${}^w\mu_g = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}; \ \frac{9}{8} = \frac{\sin\left(\frac{60° + \delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

or $\quad \sin\left(\frac{60° + \delta_m}{2}\right) = \frac{9}{8} \times \sin 30° = \frac{9}{8} \times \frac{1}{2} = \frac{9}{16} = 0.56$

or $\quad \frac{60° + \delta_m}{2} = \sin^{-1}(0.56)$

or $\quad \frac{60° + \delta_m}{2} = 34°$ or $\delta_m = 68° - 60° = 8°$

22. **(a) :** de Broglie wavelength of a moving particle, having mass m and velocity v is given by

$\lambda = \frac{h}{p} = \frac{h}{mv}$ or $m = \frac{h}{\lambda v}$

For an electron $\lambda_e = \frac{h}{m_e v_e}$ or $m_e = \frac{h}{\lambda_e v_e}$

Given : $\frac{v}{v_e} = 3$, $\frac{\lambda}{\lambda_e} = 1.813 \times 10^{-4}$

Mass of the particle, $m = m_e\left(\frac{v_e}{v}\right)\left(\frac{\lambda_e}{\lambda}\right)$

Substituting the values, we get

$$m = (9.1 \times 10^{-31}\ \text{kg}) \times \left(\frac{1}{3}\right) \times \left(\frac{1}{1.813 \times 10^{-4}}\right)$$

$$m = 1.67 \times 10^{-27}\ \text{kg}$$

23. **(a)**

24. **(c) :** Speed of the electron in the ground state of hydrogen atom is

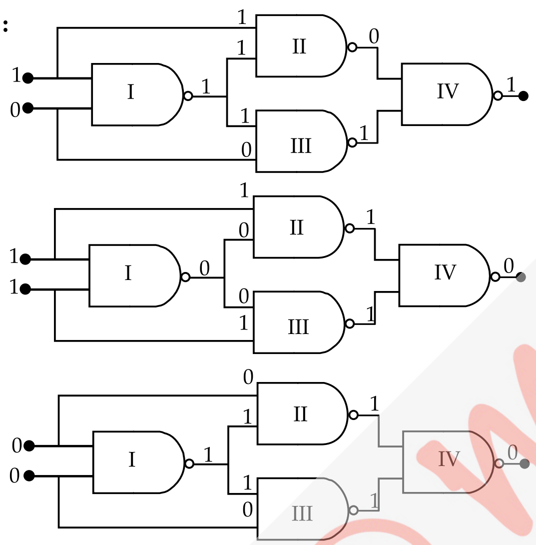
$$v = \frac{2\pi e^2}{4\pi\varepsilon_0 h} = \frac{c}{137} = c\alpha$$

where, c = speed of light in vacuum

$\alpha = \dfrac{e^2}{2\varepsilon_0 hc}$ is the fine structure constant. It is a pure number whose value is $\dfrac{1}{137}$.

$$\therefore\ \frac{v}{c} = \frac{1}{137}$$

25. **(d) :**



26. **(a) :** Here, B = 3000 G = 3000 × 10^{-4} T = 0.3 T,

A = 10 cm × 5 cm = 50 cm^2 = 50 × 10^{-4} m^2, I = 12 A

$\therefore$ IA = 12 A × 50 × 10^{-4} m^2 = 0.06 A m^2

$$I\vec{A} = 0.06\hat{i}\ \text{A m}^2, \vec{B} = 0.3\hat{k}\ \text{T}$$

Torque, $\vec{\tau} = I\vec{A} \times \vec{B} = 0.06\,\hat{i} \times 0.3\,\hat{k} = -1.8 \times 10^{-2}\,\hat{j}$ N m

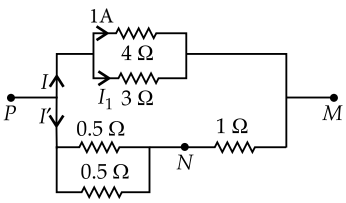
Torque = 1.8 × 10^{-2} N m. It acts along the negative y direction.

27. **(b) :** ${}^{24}_{12}\text{Mg} + {}^{4}_{2}\text{He} \rightarrow {}^{x}_{14}\text{Si} + {}^{1}_{0}n$

According to mass number conservation, we get

24 + 4 = x + 1 or x = 27

28. **(b) :**



As 4 Ω and 3 Ω are in parallel, the potential difference is same across them. If I_1 is the current through 3 Ω resistance, then 4 × 1 = 3 × I_1

or $I_1 = \dfrac{4}{3}$ A

Total resistance of 4 Ω and 3 Ω $= \dfrac{4 \times 3}{4+3} = \dfrac{12}{7}$ Ω

Total current in the upper portion of the circuit from P to $M = I = 1 + \dfrac{4}{3} = \dfrac{7}{3}$ A

Potential difference between P and M $= \dfrac{7}{3} \times \dfrac{12}{7} = 4$ V

Current in the arm PNM is

$$I' = \frac{4}{\dfrac{0.5 \times 0.5}{0.5 + 0.5} + 1} = \frac{16}{5}\ \text{A}$$

Potential difference across N and $M = I' \times 1 = \dfrac{16}{5} \times 1$

$= 3.2$ V

29. **(a) :** Here, distance between object and screen

D = 90 cm

Distance between two locations of convex lens

d = 20 cm, f = ?

$$\text{As}\ \ f = \frac{D^2 - d^2}{4D}$$

$$\therefore\ f = \frac{90^2 - 20^2}{4 \times 90} = \frac{(90+20)(90-20)}{360}$$

$$= \frac{110 \times 70}{360} = \frac{770}{36}\ \text{cm}$$

30. **(c) :** The LC circuit draws no power.

When $\omega L = \dfrac{1}{\omega C}$, the impedance of the RC and LR circuits are equal, and hence they draw the same power.

$\therefore$ $P_1 = P_2 > P_3$

31. **(d) :** Let y be the distance from the centre of sphere where the acceleration due to gravity is $a_m/3$

$\therefore$ $y = r - d$

where r is the radius of the sphere and d is the depth from the surface of the sphere.

$$\text{As}\ \ g' = g\left(1 - \frac{d}{r}\right) = g\left(\frac{r-d}{r}\right)$$

$$\therefore\ \frac{a_m}{3} = a_m\left(\frac{y}{r}\right)\ \ \text{or}\ \ y = \frac{r}{3}$$

32. **(c) :** $Y = \dfrac{Fl}{A\Delta l}$

$$2 \times 10^{11} = \frac{1 \times 10}{10^{-6}} \times \frac{1}{\Delta l} \qquad (\because F = mg)$$

$$\Delta l = \frac{10}{2 \times 10^5} = 5 \times 10^{-5}\ \text{m} = 0.05 \times 10^{-3} = 0.05\ \text{mm}$$

33. **(c) :** For adiabatic expansion

PV^γ = constant

Since $PV = RT$, hence $TV^{\gamma-1}$ = constant

$$\text{That is,}\ \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{T_1}{T_2}\right) = \frac{1}{2}$$

$$\text{Hence}\ \ \frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} < \left(\frac{1}{2}\right)$$

34. (c) : Average speed $<c> = \sqrt{\frac{8RT}{\pi M}} = 1.6\sqrt{\frac{RT}{M}}$

$\therefore$ temperature of both the sample is same.

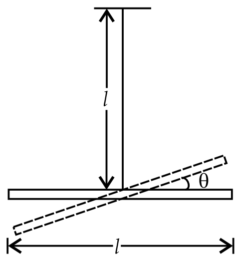
$\therefore \quad <c_{\mathrm{H}}> = 1.6\sqrt{\frac{RT}{1}} \quad \therefore \quad <c_{\mathrm{He}}> = 1.6\sqrt{\frac{RT}{4}}$

$\Rightarrow \quad <c_{\mathrm{H}}> = 2 <c_{\mathrm{He}}>$

35. (c) : $\frac{K_1 \pi r^2 \Delta\theta t}{l} + K_2 \frac{\pi\left[(2r)^2 - r^2\right]\Delta\theta t}{l} = \frac{K\pi(2r)^2\Delta\theta t}{l}$

$K = \frac{K_1 + 3K_2}{4}$

36. (b) : $T = 2\pi\sqrt{\frac{I}{mgl}} = \sqrt{\frac{ml^2/12}{mgl}} = \sqrt{\frac{l}{12g}}$



37. (b) : $\frac{\lambda}{4} = 32 + 0.3d$ ...(i)

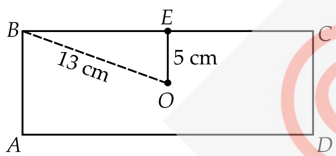
[d = diameter of the tube]

$\frac{3\lambda}{4} = 100 + 0.3d$ ...(ii)

From (i) and (ii), we get $\frac{\lambda}{2} = 68$ cm

$\frac{\lambda}{4} = 34$ cm or $0.3d = 2$ cm.

38. (a) :



$W = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$

$= 100 \times 0.104 \times 10^{-12} \times 9 \times 10^9 \left[\frac{1}{0.05} - \frac{1}{0.13}\right]$.

$= 1.04 \times \frac{72}{65} = 1.152$ J

39. (a) : For achromatic combination

$\frac{\omega_1}{f_1} = \frac{-\omega_2}{f_2}$ or $\omega_1 P_1 = -\omega_2 P_2$

or $P_2 = -P_1 = -2\text{D}$ $\quad (\because \omega_1 = \omega_2)$

Focal length $= \frac{1}{P_2} = \frac{-1}{2} = -0.5$ m

or focal length = – 50 cm.

40. (a) : Let f_o and f_e be the focal lengths of the objective and eyepiece respectively. For normal ajustment, distance between the objective to the eyepiece (tube length) $= f_o + f_e$. Treating the line on the objective as the object, and the eyepiece as the lens, $u = -(f_o + f_e)$ and $f = f_e$.

$\frac{1}{v} - \frac{1}{-(f_o + f_e)} = \frac{1}{f_e}$ or $\frac{1}{v} = \frac{1}{f_e} - \frac{1}{(f_o + f_e)} = \frac{f_o}{(f_o + f_e)f_e}$

or $v = \frac{(f_o + f_e)f_e}{f_o}$.

Magnification $= \left|\frac{v}{u}\right| = \frac{f_e}{f_o} = \frac{\text{image size}}{\text{object size}} = \frac{l}{L}$.

$\therefore \frac{f_o}{f_e} = \frac{L}{l} =$ magnification of telescope in normal adjustment.

41. (d) : In Rutherford's experiment, beam of helium nuclei *i.e.*, α-particles was used.

42. (c) : Volume of 1 mole of an ideal gas at 273 K and 1 atm is 22.4 L.

Volume at 373 K and 1 atm will be given by

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ (as P = constant)

i.e., $\frac{22.4}{273} = \frac{V_2}{373}$ or $V_2 = 30.6$ L.

43. (d) : $E^\circ_{\mathrm{Co^{2+}/Co}} = -0.28$ V, $E^\circ_{\mathrm{Ag^+/Ag}} = 0.80$ V

Hence, cobalt is anode, *i.e.*, oxidation takes place on cobalt electrode. Cell reaction is

$\mathrm{Co} + 2\mathrm{Ag}^+ \rightarrow \mathrm{Co}^{2+} + 2\mathrm{Ag}$

$\therefore \quad E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{RT}{nF}\ln\frac{[\mathrm{Co}^{2+}]}{[\mathrm{Ag}^+]^2}$

Less is the factor $[\mathrm{Co}^{2+}]/[\mathrm{Ag}^+]^2$, greater is the E_{cell}.

44. (d)

45. (b) : (i) $AB_{(s)} + aq \rightarrow AB_{(aq)}$; $\Delta H = -21$ J

(ii) $AB_{(s)} + x\mathrm{H_2O} \rightarrow AB \cdot x\mathrm{H_2O}_{(s)}$; $\Delta H = -29.4$ J

$AB \cdot x\mathrm{H_2O}_{(s)} + aq \rightarrow AB_{(aq)}$; $\Delta H = ?$

Equation (i) is equivalent to

$AB_{(s)} + x\mathrm{H_2O} \rightarrow AB \cdot x\mathrm{H_2O}_{(s)}$; $\Delta H = \Delta H_1$

$AB \cdot x\mathrm{H_2O}_{(s)} + aq \rightarrow AB_{(aq)}$; $\Delta H = \Delta H_2$

$\Delta H_1 + \Delta H_2 = -21$

$-29.4 + \Delta H_2 = -21$

$\therefore \quad \Delta H_2 = -21 + 29.4 = 8.4$ J mol^{-1}.

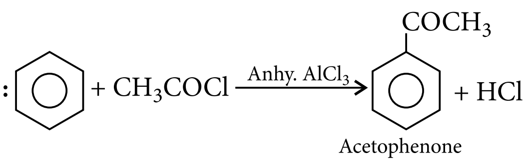
46. (d) : Let $A_2X_3 \rightleftharpoons 2A^{3+} + 3X^{2-}$

$K_{sp} = [A^{3+}]^2 [X^{2-}]^3$

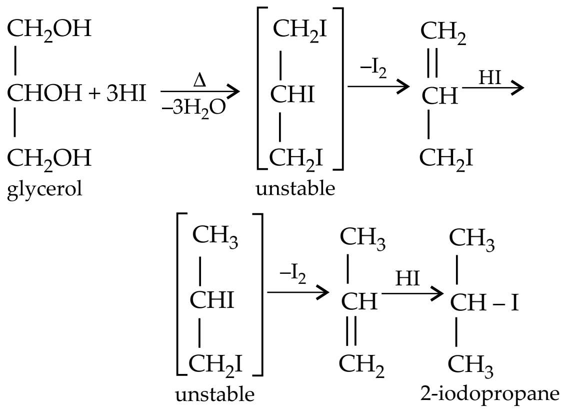
Since solubility = y mol dm^{-3}

$K_{sp} = (2y)^2 (3y)^3 = 108\, y^5$.

47. (d) :



48. **(c) :**



49. **(d) :** Carbocation (ii) is most stable due to resonance.

$$\underset{\text{(ii)}}{CH_3-\overset{+}{C}H-\ddot{O}-CH_3} \longleftrightarrow CH_3-CH=\overset{+}{\ddot{O}}-CH_3$$

Carbocation (iii) is least stable due to electron withdrawing effect of the adjacent carbonyl group while carbocation (i) is less stable than (ii) because it is only stabilised by the +*I*-effect of the two CH_3 groups. Thus, order of stability is: (ii) > (i) > (iii).

50. **(a) :** Two given compounds are enantiomers *i.e.*, non-superimposable mirror images of each other which rotate the plane polarised light by same angle but in opposite direction *i.e.*, if one rotates by +52° then another compound rotates by –52°.

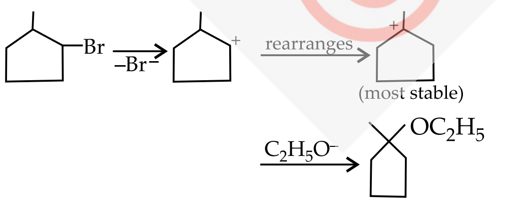
51. **(d) :** For *bcc*, $d = \frac{\sqrt{3}}{2}a$

$$a = \frac{2d}{\sqrt{3}} = \frac{2\times 4.52}{1.732} = 5.219\ \text{Å} = 522\ \text{pm}$$

$$\rho = \frac{Z\times M}{a^3\times N_0\times 10^{-30}} = \frac{2\times 39}{(522)^3\times 6.023\times 10^{23}\times 10^{-30}}$$

$= 0.91\ \text{g/cm}^3 = 910\ \text{kg m}^{-3}$.

52. **(a) :**



53. **(b) :**

$$NH_2-\underset{\underset{O}{\|}}{C}-NH_2 + SOCl_2 \longrightarrow \underset{\text{(cyanamide)}}{NH_2-C\equiv N} + SO_2 + 2HCl$$

54. **(a) :** Degree of unsaturation

$$= \frac{2n_1 - n_2 + 2}{2} = \frac{2\times 7 - 6 + 2}{2} = 5$$

55. (c) **56.** (a)

57. **(c) :** More is the no. of ions, more is the conductance:
$[Pt(NH_3)_6]Cl_4 > [Pt(NH_3)_5Cl]Cl_3 > [Pt(NH_3)_4Cl_2]Cl_2 > [Pt(NH_3)_3Cl_3]Cl$

58. (a)

59. **(b) :** $Fe^{3+} + 3\overset{-}{SCN} \longrightarrow \underset{\text{(red ppt.)}}{Fe(SCN)_3\downarrow}$

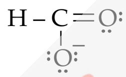
60. **(a) :** In Tischenko reaction in presence of aluminium ethoxide, aliphatic aldehydes undergo self oxidation and reduction and forms acid and alcohol.

$$2CH_3CHO \xrightarrow{Al(OC_2H_5)_3} CH_3COOH + C_2H_5OH$$
$$\downarrow$$
$$CH_3COOC_2H_5$$

61. (d)

62. **(b) :** Rate of diffusion of hydrogen is more than CO_2.

63. **(c) :** Formate ion, HCO_2^- has 1 + 4 + 2 × 6 +1 *i.e.*, 18 electrons. Its Lewis structure is which has no lone pair of electrons on the carbon atom



64. **(c) :** $\Delta T = K_f m$

or $6 = \frac{1.86\times w}{60}$ or $w = 193.5$ g.

65. **(c) :** Diffusion through animal membrane does not result in sol destruction.

66. **(d) :** Cotton is cellulose $(C_6H_{12}O_6)_x$. When it comes in contact with conc. H_2SO_4 which has a dehydrating property, it removes the elements of H_2O from cotton leaving only black carbon in the form of charred particles.

$$C_6H_{12}O_6 \xrightarrow{H_2SO_4} \underset{\substack{\text{Charred}\\ \text{Particle}}}{6C} + 6H_2O$$

67. **(b) :** Cu^{2+} gives iodometric titration as it gives I_2 on reacting with KI.
$Cu^{2+} + 4I^- \rightarrow 2CuI + I_2$

68. **(d) :** Peaks represent the highest values. Since noble gases have highest ionisation enthalpy values in their respective periods, these occupy peak positions in the ionisation enthalpy curve.

69. **(c) :** Np and Pu in NpO_3^+ and PuO_3^+ oxocations show +7 oxidation states which are not so stable.

70. **(c) :** Fe is sp^3d^2 hybridised in $[Fe(H_2O)_5NO]^{2+}$ complex.

71. **(d) :** As the molecular mass increases, the hydrocarbon part increases and the extent of H-bonding decreases and thus the solubility decreases, *i.e.*, option (d) is correct.

72. (b)

73. **(b) :** D_2O has higher (3.8°C) freezing point than H_2O (0°C).

74. **(b) :** $\overset{-1}{H_2O_2} + \overset{+4}{ClO_2} + OH^- \rightarrow Cl^- + \overset{0}{O_2} + 6H_2O$

(i) $\overset{-1}{H_2O_2} \rightarrow \overset{0}{O_2} \uparrow 2(-2\rightarrow 0)$

(ii) $\overset{+4}{ClO_2} \rightarrow Cl^- \downarrow 5(+4 \rightarrow -1)$

To make increase or decrease in O.No. equal, multiply eqn. (i) by 5 and eqn. (ii) by 2 and add. Hence

$5H_2O_2 + 2ClO_2 \rightarrow 5O_2 + 2Cl^-$

To balance O atoms, add $4H_2O$ on RHS. Hence

$5H_2O_2 + 2ClO_2 \rightarrow 5O_2 + 2Cl^- + 4H_2O$

To balance H atoms, add $2OH^-$ on LHS and $2H_2O$ on RHS. Hence

$5H_2O_2 + 2ClO_2 + 2OH^- \rightarrow 5O_2 + 2Cl^- + 6H_2O$

75. **(d):** Non-metallic, Z = 35 (bromine)
Metallic, Z = 80 (mercury)

76. (c) : pH's : Milk = 6.8, Black coffee = 5.0, Human tears = 7.4, Lemon juice = 2.2.

77. **(c):** By Henry's law ($m \propto p$)

78. **(b):** Each enzyme can catalyse only one reaction. Thus, statement (b) is wrong.

79. **(d):** $q + w = \Delta U$ is a state function.

80. **(d):** Inert gases like He, Ne, Ar etc. normally do not enter into chemical bonding and hence their valency is zero.

81. (b)	**82. (a)**	**83. (a)**	**84. (b)**	**85. (b)**
86. (d)	**87. (a)**	**88. (c)**	**89. (c)**	**90. (d)**
91. (b)	**92. (b)**	**93. (d)**	**94. (a)**	**95. (d)**

96-97 : Clearly, S must be released on fifth Friday. P must be released a week before T *i.e.*, order PT must be followed. But T cannot be released in the last. Also, Q must be released immediately after U *i.e.*, order UQ must be followed. But Q cannot precede S. So, U and Q can be released on first and second Fridays respectively and P and T on third and fourth Fridays respectively. R, which cannot be released on second Friday, shall be released last.
Thus, the order followed will be : U, Q, P, T, S, R.

96. **(a):** Clearly, the release of P precedes that of T.

97. **(a):** P is released immediately after Q.

98. **(c):** The last four letters of the word are written in the reverse order, followed by the first four letters in the same order. In the group of letters so obtained, each of the first four letters is moved one step backward while each of the last four letters is moved one step forward to get the code. Thus, we have :

SHOULDER → SHOU/LDER → REDL/SHOU → QDCK/TIPV

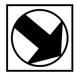
99. **(b):** All the letters of the word, except the last letter, are written in the reverse order and in the group of letters so obtained, each letter is moved two steps forward to get the code. Thus, we have :

AVOID → IOVAD → KQXCF

100. **(d) :** Using the correct symbols, we have:
Given expression = 27 + 81 ÷ 9 – 6 = 27 + 9 – 6 = 36 – 6 = 30.

101. **(d) :** Using the correct symbols, we have:
Given expression = 24 × 12 + 18 ÷ 9 = 288 + 2 = 290.

102. **(d):** Clearly, fig. (A) rotates through 135°CW to form fig. (B). Similar relationship will give fig. (D) from fig. (C). Hence, fig. (d) is the answer.



103. **(d):** Fig. (A) is divided into as many parts as the number of sides in the figure, to get fig. (B). Similarly, fig. (D) will be obtained when fig. (C) is divided into as many parts as the number of sides in fig. (C). Hence, fig. (d) is the answer.

104. **(d):** The three scattered elements in fig. I are arranged in a single row to get fig. II.

105. **(d):** In all other pairs, 2nd number = (1st number × 3) + 1.

106. **(a) :** Given $|z+1| < |z-2|$ and $\omega = 3z + 2 + i$

$\therefore \quad \omega + \bar{\omega} = 3z + 2 + i + 3\bar{z} + 2 - i$

$\therefore \quad \omega + \bar{\omega} = 3(z + \bar{z}) + 4$ ...(i)

Now $|z+1|^2 < |z-2|^2$

$(z+1)(\bar{z}+1) < (z-2)(\bar{z}-2) \Rightarrow z + \bar{z} < \circledcirc$ ...(ii)

from (i) and (ii), we get

$$\frac{\omega + \bar{\omega} - 4}{3} < 1 \Rightarrow \omega + \bar{\omega} < 7$$

107. **(b) :** Let the circle $|z - z'| = c$, which touches the circles $|z - z_1| = a$ and $|z - z_2| = b$ externally, then distance between centres = Sum of radii

$\therefore \quad |z' - z_1| = c + a$ ...(i)

and $|z' - z_2| = c + b$ ...(ii)

Subtracting (i) and (ii) we get

$|z' - z_1| - |z' - z_2| = a - b$

Hence locus of the centre of the assuming circle is

$|z - z_1| - |z - z_2| = a - b$

but $a - b < a + b$. Hence locus of the centre of the assuming circle is a hyperbola.

108. **(d) :** $x^2 + x + 1 = 0$

$\Rightarrow \quad (x - \omega)(x - \omega^2) = 0 \quad \therefore x = \omega, \omega^2$

$\therefore \quad \alpha = \omega, \beta = \omega^2 \quad (\because \omega, \omega^2$ are the cube roots of unity$)$

$\alpha^3 = 1, \beta^3 = 1, \alpha\beta = 1$

$\alpha^{19} = (\alpha^3)^6 \alpha = 1^6 \alpha = \alpha = \omega$

$\beta^7 = \beta^6 . \beta = 1^2 . \beta = \beta = \omega^2$

$\Rightarrow \quad \alpha^{19} + \beta^7 = \omega + \omega^2 = -1$ & $\alpha^{19} . \beta^7 = \omega.\omega^2 = \omega^3 = 1$

$\therefore \quad$ Equation whose roots are α^{19} & β^7, is

$x^2 - (\alpha^{19} + \beta^7)x + \alpha^{19}\beta^7 = 0 \Rightarrow x^2 + x + 1 = 0$

109. (c) : Given equation is

$(3-x)^4 + (2-x)^4 = (5-2x)^4$ ...(i)

Let $a = 3 - x$ & $b = 2 - x$

$\therefore$ Equation (i) becomes $a^4 + b^4 = (a+b)^4$

$\Rightarrow a^4 + b^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$\Rightarrow 4a^3b + 6a^2b^2 + 4ab^3 = 0$

$\Rightarrow 2ab\,(2a^2 + 3ab + 2b^2) = 0$

$\therefore a = 0 \Rightarrow 3 - x = 0 \Rightarrow x = 3$

$b = 0 \Rightarrow 2 - x = 0 \Rightarrow x = 2$ and $2a^2 + 3ab + 2b^2 = 0$

$$\Rightarrow 2\left(\frac{a}{b}\right)^2 + 3\left(\frac{a}{b}\right) + 2 = 0$$

which is a quadratic $\left(\frac{a}{b}\right)$

$$\Rightarrow \frac{3-x}{2-x} = \frac{-3 \pm i\sqrt{7}}{4}$$

$$\Rightarrow \frac{3-x}{2-x} - 1 = \frac{-7 \pm i\sqrt{7}}{4} \qquad \Rightarrow \frac{1}{2-x} = \frac{-7 \pm i\sqrt{7}}{4}$$

$$\Rightarrow 2 - x = \frac{4}{-7 \pm i\sqrt{7}} \qquad \Rightarrow x = 2 - \frac{4}{-7 \pm i\sqrt{7}}$$

$$\Rightarrow x = \frac{-18 \pm 2i\sqrt{7}}{-7 \pm i\sqrt{7}}$$

Taking '+' sign, we get

$$x = \frac{(-18 + 2i\sqrt{7})(-7 - i\sqrt{7})}{(-7 + i\sqrt{7})(-7 - i\sqrt{7})}$$

$$\Rightarrow x = \frac{140 + 4i\sqrt{7}}{56} \qquad \therefore x = \frac{35 + i\sqrt{7}}{14}$$

Taking ' – ' sign, we get

$$x = \frac{35 - i\sqrt{7}}{14}$$

Hence, roots of (i) are

$$x = 2, 3, \frac{35 \pm i\sqrt{7}}{14}$$

i.e., Two real and two imaginary.

110. (a) : $\because (1 + x + 2x^2)^{20} = a_0 + a_1 x + a_2 x^2 + \ldots + a_{40} x^{40} +$

Putting $x = 1$ & -1 we get

$4^{20} = a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + \ldots$
$+ a_{37} + a_{38} + a_{39} + a_{40}$ ...(i)

$2^{20} = a_0 - a_1 + a_2 - a_3 + a_4 - a_5 + \ldots$
$- a_{37} + a_{38} - a_{39} + a_{40}$ ...(ii)

Subtracting (i) and (ii), we get

$4^{20} - 2^{20} = 2\,(a_1 + a_3 + a_5 + \ldots + a_{37} + a_{39})$

$\therefore a_1 + a_3 + a_5 + \ldots + a_{37} = 2^{19}\,(2^{20} - 1) - a_{39}$ ...(iii)

$\therefore a_{39}$ = coefficient of x^{39} in $(1 + x + 2x^2)^{20}$

= coefficient of x^{39} in $(1 + x\,(1 + 2x))^{20}$ ...(iv)

General term in $(1 + x\,(1 + 2x))^{20}$

$T_{r+1} = {}^{20}C_r x^r\,(1 + 2x)^r$

Put $r = 20 \Rightarrow T_{21} = {}^{20}C_{20}\,x^{20}\,(1 + 2x)^{20}$

$= x^{20}(1 + {}^{20}C_1\,(2x) + {}^{20}C_2\,(2x)^2 + \ldots + {}^{20}C_{19}(2x)^{19}$
$+ {}^{20}C_{20}\,(2x)^{20})$

$\therefore$ Coefficient of $x^{39} = {}^{20}C_{19}\,2^{19} = 20.2^{19} = a^{39}$

$\therefore$ From (iii)

$a_1 + a_3 + a_5 + \ldots + a_{37} = 2^{19}\,(2^{20} - 21).$

111. (a) : We can write Δ as

$$\Delta = \begin{vmatrix} 0 & \sin x - \cos x & \log\tan x \\ -(\sin x - \cos x) & 0 & -\log\tan x \\ -\log\tan x & \log\tan x & 0 \end{vmatrix}$$

$$= (-1)^3 \begin{vmatrix} 0 & -(\sin x - \cos x) & -\log\tan x \\ \sin x - \cos x & 0 & \log\tan x \\ \log\tan x & -\log\tan x & 0 \end{vmatrix}$$

[taking – 1 common from R_1, R_2 and R_3]

$= -\Delta$ [using reflection property] $\Rightarrow 2\Delta = 0 \Rightarrow \Delta = 0.$

112. (d) : $2\left[\frac{x}{8}\right]^2 + 3\left[\frac{x}{8}\right] - 20 = 0$

$$\Rightarrow \left[\frac{x}{8}\right] = \frac{5}{2} \text{ or } -4$$

As $\left[\frac{x}{8}\right]$ is an integer, we take

$$\left[\frac{x}{8}\right] = -4 \Rightarrow -4 \le \frac{x}{8} < -3$$

$\Rightarrow -32 \le x < -24$. Thus, $a = -32$, and $b = -24$.

Therefore, $b - a = 8$.

113. (c) : We have $\sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-i}$

= the number of ways of choosing m $(0 \le m \le 10)$ persons out of 10 men and 20 women

= the number of ways of choosing m $(0 \le m \le 10)$ out of 30 persons $= {}^{30}C_m$

But ${}^{30}C_m$ is maximum for $m = 15$.

114. (b) : We have $\tan^2 x - \tan^4 x + \tan^6 x - \tan^8 x + \ldots$

$$= \frac{\tan^2 x}{1 - (-\tan^2 x)} = \frac{\tan^2 x}{\sec^2 x} = \sin^2 x$$

$\therefore y = \exp\{(\tan^2 x - \tan^4 x + \tan^6 x - \tan^8 x + \ldots)\log_e 16\}$

$= \exp\{(\sin^2 x)\log_e 16\} = \exp\{\log_e(16^{\sin 2x})\} = 16^{\sin 2\,x}$

As y satisfies $x^2 - 3x + 2 = 0$, we get $y = 1$ or $y = 2$

$\Rightarrow 16^{\sin 2\,x} = 1$ or $16^{\sin 2\,x} = 2$

Since $0 < x < \pi/4$, $0 < \sin x < 1/\sqrt{2}$

$\Rightarrow 0 < \sin^2 x < 1/2 \;\therefore 16^{\sin 2\,x} = 1$ is not possible.

Thus, $16^{\sin 2\,x} = 2 \Rightarrow \sin^2 x = 1/4$

Thus, $\cos^2 x + \cos^4 x = (1 - \sin^2 x) + (1 - \sin^2 x)^2 = 21/16$

115. (a) : The given expression is equal to

$\sin^2\alpha + \cos(\alpha + \beta)\,[\cos(\alpha + \beta) + 2\sin\alpha\sin\beta]$

$= \sin^2\alpha + \cos(\alpha + \beta)\,[\cos\alpha\cos\beta + \sin\alpha\sin\beta]$

$= \sin^2\alpha + \cos(\alpha + \beta)\cos(\alpha - \beta)$

$= \sin^2\alpha + \cos^2\alpha - \sin^2\beta = 1 - \sin^2\beta = \cos^2\beta$

which is independent of α only.

116. (b) : We have $f(x) = (x - 1)(x - 2)(x - 3)$.

Since $1, 2, 3 \in R$ and $f(1) = 0, f(2) = 0, f(3) = 0$, f cannot be one – one.

Let $k \in R$. $f(x) = k$ implies $(x - 1)(x - 2)(x - 3) = k$.

$\Rightarrow x^3 - 6x^2 + 11x - 6 - k = 0.$

Since a cubic equation has at least one real root, there exist some $x' \in R$ such that $f(x') = k$. $\therefore f$ is onto.

117. (a) : $\sin nx = \sin(nx + 2\pi) = \sin n\left(x + \frac{2\pi}{n}\right)$

$\therefore$ Period of $\sin nx = 2\pi/n$

Also, $\sin\frac{x}{n} = \sin\left(\frac{x}{n} + 2\pi\right) = \sin\frac{1}{n}(x + 2n\pi)$

$\therefore$ Period of $\sin\frac{x}{n} = 2n\pi$

L.C.M. of $\frac{2\pi}{n}$, $2n\pi = 2n\pi$

$\therefore$ Period of $f(x) = 2n\pi$. $2n\pi = 4\pi \Rightarrow n = 2$.

118. (a) : $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$

$$= \frac{1 - \cos 2x}{2} + \frac{1 - \cos(2x + 2\pi/3)}{2} + \frac{1}{2}\left[\cos\left(2x + \frac{\pi}{3}\right) + \cos\frac{\pi}{3}\right]$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2x + \frac{1}{2} - \frac{1}{2}\cos\left(2x + \frac{2\pi}{3}\right) + \frac{1}{2}\cos\left(2x + \frac{\pi}{3}\right) + \frac{1}{2}\left(\frac{1}{2}\right)$$

$$= \frac{5}{4} - \frac{1}{2}\cos 2x - \frac{1}{2}\cos\left(2x + \frac{2\pi}{3}\right) + \frac{1}{2}\cos\left(2x + \frac{\pi}{3}\right)$$

$$= \frac{5}{4} - \frac{1}{2}\left(\cos\left(2x + \frac{2\pi}{3}\right) + \cos 2x\right) + \frac{1}{2}\cos\left(2x + \frac{\pi}{3}\right)$$

$$= \frac{5}{4} - \frac{1}{2}\cdot 2\cos\left(2x + \frac{\pi}{3}\right)\cos\frac{\pi}{3} + \frac{1}{2}\cos\left(2x + \frac{\pi}{3}\right) = \frac{5}{4}$$

$\therefore$ $(gof)(x) = g(f(x)) = g(5/4) = 1$.

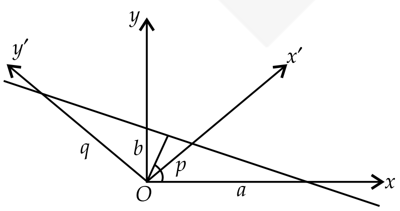
119. (b) : It is clear from the figure that the perpendicular distance of the line from the origin will remain constant when the axes are rotated through an angle θ. Let α be the angle between the perpendicular OL (such that $OL = r$) and the x-axis before rotation. We have

$r = a\cos\alpha$, $r = b\sin\alpha$

$\Rightarrow \left(\frac{r}{a}\right)^2 + \left(\frac{r}{b}\right)^2 = 1 \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{r^2}$ ...(i)

If the axes are rotated through an angle θ, then

$r = p\cos(\alpha - \theta)$, $r = q\sin(\alpha - \theta)$



$\Rightarrow \left(\frac{r}{p}\right)^2 + \left(\frac{r}{q}\right)^2 = 1$

$\Rightarrow \frac{1}{p^2} + \frac{1}{q^2} = \frac{1}{r^2}$ ...(ii)

By (i) and (ii), we have $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$.

120. (c) : $x = t^2 + t + 1$, $y = t^2 - t + 1$ ...(i)

Eliminating the parameter t from (i) as follows :

$x - y = 2t$

$\therefore$ $x = \left(\frac{x-y}{2}\right)^2 + \left(\frac{x-y}{2}\right) + 1$

$\Rightarrow 4x = (x - y)^2 + 2x - 2y + 4$

$\Rightarrow (x - y)^2 = 2(x + y - 2)$; which is a equation of parabola.

121. (a) : Let midpoint of the chord be (h, k), then equation of the chord is $\frac{hx}{a^2} + \frac{ky}{b^2} - 1 = \frac{h^2}{a^2} + \frac{k^2}{b^2} - 1$

$\Rightarrow y = -\frac{b^2}{a^2}\cdot\frac{h}{k}x + \left(\frac{h^2}{a^2} + \frac{k^2}{b^2}\right)\frac{b^2}{k}$ ...(i)

Since line (i) is touching the circle $x^2 + y^2 = c^2$ ...(ii)

$\therefore \left(\frac{h^2}{a^2} + \frac{k^2}{b^2}\right)^2 \frac{b^4}{k^2} = c^2\left(1 + \frac{b^4}{a^4}\frac{h^2}{k^2}\right)$

$\therefore$ Required locus is $(b^2x^2 + a^2y^2)^2 = c^2(b^4x^2 + a^4y^2)$.

122. (a) : Let coordinates of pole be (h, k), then equation of the polar of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$\frac{hx}{a^2} + \frac{ky}{b^2} = 1 \Rightarrow y = -\frac{b^2}{a^2}\frac{h}{x}x + \frac{b^2}{k}$ ...(i)

Since line (i) is touching the circle $x^2 + y^2 = c^2$

$\therefore \frac{b^4}{k^2} = c^2\left(1 + \frac{b^4}{a^4} + \frac{h^2}{k^2}\right)$

$\therefore$ Required locus is $c^2(b^4x^2 + a^4y^2) = a^4b^4$.

123. (c) : We have $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = 3\pi/2$

it is possible only when

$\sin^{-1}x = \pi/2 \Rightarrow x = 1$, $\sin^{-1}y = \pi/2 \Rightarrow y = 1$

$\sin^{-1}z = \pi/2 \Rightarrow z = 1$

$\therefore$ $x^{100} + y^{100} + z^{100} - \frac{3}{x^{101} + y^{101} + z^{101}}$

$= 1 + 1 + 1 - 3/3 = 3 - 1 = 2$

124. (a) : We have

$$\tan^{-1}\left(\frac{c_1x - y}{c_1y + x}\right) + \tan^{-1}\left(\frac{c_2 - c_1}{1 + c_2c_1}\right) + \tan^{-1}\left(\frac{c_3 - c_2}{1 + c_2c_3}\right) + \ldots + \tan^{-1}\left(\frac{1}{c_n}\right)$$

$$\tan^{-1}\left(\frac{\frac{x}{y} - \frac{1}{c_1}}{1 + \frac{x}{y}\frac{1}{c_1}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_1} - \frac{1}{c_2}}{1 + \frac{1}{c_1c_2}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_2} - \frac{1}{c_3}}{1 + \frac{1}{c_2c_3}}\right) + \ldots + \tan^{-1}\left(\frac{1}{c_n}\right)$$

$$= \tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{1}{c_1}\right) + \tan^{-1}\left(\frac{1}{c_1}\right) - \tan^{-1}\left(\frac{1}{c_2}\right) + \tan^{-1}\left(\frac{1}{c_2}\right) - \tan^{-1}\left(\frac{1}{c_3}\right) + \ldots - \tan^{-1}\left(\frac{1}{c_n}\right) + \tan^{-1}\left(\frac{1}{c_n}\right)$$

$$= \tan^{-1}\left(\frac{x}{y}\right)$$

125. (d) : Let X denote the largest integer out of the m chosen integers.

Thus, $P(X \le k) = \left(\frac{k}{n}\right)^m$ [Integers are chosen with replacement]

and $P(X \ge k-1) = \left(\frac{k-1}{n}\right)^m$

Hence, $P(X = k) = P(X \le k) - P(X \le k - 1)$

$$= \left(\frac{k}{n}\right)^m - \left(\frac{k-1}{n}\right)^m.$$

126. (d) : For any arrangement of the 7 persons (other than A, B, C), the three persons A, B and C can be arranged in 3! ways out of which there is only one way in which A comes before B and B comes before C.
Hence, required probability is (1/3)!.

127. (b) : The given system of equation can be written as

$$\begin{bmatrix} 1/a^2 & 1/b^2 & -1/c^2 \\ 1/a^2 & -1/b^2 & 1/c^2 \\ -1/a^2 & 1/b^2 & 1/c^2 \end{bmatrix}\begin{bmatrix} x^2 \\ y^2 \\ z^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{vmatrix} 2/a^2 & 0 & 0 \\ 0 & 0 & 2/c^2 \\ -1/a^2 & 1/b^2 & 1/c^2 \end{vmatrix}$$

$$\left[\text{Applying } R_1 \to R_1 + R_2 \text{ and } R_2 \to R_2 + R_3\right]$$

$$= \frac{2}{a^2} \times \frac{-2}{b^2c^2} = \frac{-4}{a^2b^2c^2} \ne 0$$

Hence, the given system of equations has a unique solution.

128. (c) : We have

$f(x) = (x^2 - 1)^n g(x)$ where $g(1) \ne 0$

and $f'(x) = 2nx(x^2 - 1)^{n-1} g(x) + (x^2 - 1)^n g'(x)$

$= (x^2 - 1)^{n-1} [2nx\, g(x) + (x^2 - 1)\, g'(x)]$

$= (x + 1)^{n-1} (x - 1)^{n-1} h(x)$

Now, $h(1) = 2n\, g(1) \ne 0$. Thus, $f'(x)$ vanishes at $x = \pm 1$.
Now, $f'(x)$ will change sign at $x = 1$ only if $n - 1$ is odd, *i.e.*, n is even.

129. (d) : We have $f(x) = 2\log x - \log_x (0.01)$, $x > 1$

$$= 2\log x - \frac{\log 10^{-2}}{\log x} = 2\left(\log x + \frac{1}{\log x}\right)$$

$$\ge 2\left(2\sqrt{\log x \cdot \frac{1}{\log x}}\right) = 4 \qquad \left[\text{using } a + b \ge \sqrt{ab}\right]$$

Hence, least value = 4.

130. (a) : We have $f(x) = \begin{cases} x^2 - 2x - 2, & 0 \le x < 3 \\ 3 - x, & 3 \le x \le 5 \end{cases}$

and $f'(x) = \begin{cases} 2x - 2, & 0 \le x < 3 \\ -1, & 3 \le x \le 5 \end{cases}$

Now, $2x - 2 < 0\ \forall\ x \in (0, 1)$ and $> 0\ \forall\ x \in (1, 3)$
$\Rightarrow f'(x)$ strictly decreases in (0, 1), strictly increases in (1, 3) & strictly decreases in (3, 5)
The least value = –3
Greatest value does not exist
$x = 0$ is point of maxima
$x = 3$ is neither a point of maxima nor a minima.

131. (c) : Putting $x = z^6$ and $dx = 6z^5 dz$, we have

$$I = \int \frac{dx}{\sqrt{x} - \sqrt[3]{x}} = \int \frac{6z^5 dz}{z^3 - z^2} = \int \frac{6z^3 dz}{z - 1} = 6\int \frac{(z^3 - 1)dz}{z - 1} + 6\int \frac{dz}{z - 1}$$

$$= 6\int (z^2 + z + 1)dz + 6\int \frac{dz}{z - 1}$$

$$= 2z^3 + 3z^2 + 6z + 6 \ln | z - 1 | + C$$

$$= 2x^{1/2} + 3x^{1/3} + 6x^{1/6} + 6 \ln | x^{1/6} - 1 | + C.$$

132. (c) : Putting $2^{2^{2^x}} = z$ and $2^{2^{2^x}}.2^{2^x}.2^x(\ln 2)^3 dx = dz$,
We have

$$I = \int 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x dx = \int \frac{dz}{(\ln 2)^3} = \left[\frac{z}{(\ln 2)^3} + C\right] = \frac{2^{2^{2^x}}}{(\ln 2)^3} + C.$$

133. (d) : We have

$$f(x) = \frac{1}{x^2}\int_2^x [t^2 + f'(t)]dt$$

Differentiating w.r.t. x, we have

$$f'(x) = \frac{1}{x^2}[x^2 + f'(x)] + \frac{-2}{x^3}\int_2^x [t^2 + f'(t)]dt$$

Hence, we have

$$f'(2) = \frac{1}{4}[4 + f'(2)] + \frac{-2}{8}\int_2^2 [t^2 + f'(t)]dt = 1 + \frac{1}{4}f'(2) + 0$$

i.e. $f'(2) = 4/3$.

134. (a) : We have

$$\text{L.H.D.} = \lim_{h \to 0} \frac{f(x) - f(x - h)}{h}$$

$$= \lim_{h \to 0} \frac{[k]\sin k\pi - [k - h]\sin(k - h)\pi}{h}$$

$$= \lim_{h \to 0} -(k - 1)\frac{\sin(k - h)\pi}{h}\left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{h \to 0} -(k - 1)\frac{-\pi\cos(k - h)\pi}{1}$$

$$= (k - 1)\pi\,(-1)^k.$$

135. (b) : We have

$$y = \cot^{-1}\left(\frac{\ln(e/x^2)}{\ln(ex^2)}\right) + \cot^{-1}\left(\frac{\ln(ex^4)}{\ln(e^2/x^2)}\right)$$

$$= \tan^{-1}\left(\frac{1 + \ln x^2}{1 - \ln x^2}\right) + \tan^{-1}\left(\frac{2 - \ln x^2}{1 + 2\ln x^2}\right)$$

$$= \tan^{-1} 1 + \tan^{-1}(\ln x^2) + \tan^{-1} 2 - \tan^{-1}(\ln x^2)$$

$$= \tan^{-1} 1 + \tan^{-1} 2.$$

Hence, $\dfrac{d^2y}{dx^2} = 0.$

136. (a) : We have $\int_a^x f(t)dt - \int_0^y g(t)dt = b$

Differentiating w.r.t. x, we have

$$f(x) - g(y)\frac{dy}{dx} = 0, \text{ gives } \frac{dy}{dx} = \frac{f(x)}{g(y)}$$

Hence, $\left(\dfrac{dy}{dx}\right)_{(x_0, y_0)} = \dfrac{f(x_0)}{g(y_0)}.$

137. (b) : We have

$$f(x)=\frac{\ln(x+a)}{\ln(x+b)},(a,b>0)$$

and $$f'(x)=\frac{\frac{\ln(x+b)}{x+a}-\frac{\ln(x+a)}{x+b}}{\ln^2(x+b)}$$

$$=\frac{(x+b)\ln(x+b)-(x+a)\ln(x+a)}{(x+a)(x+b)\ln^2(x+b)}$$

$$=\frac{\ln(x+b)^{x+b}-\ln(x+a)^{x+a}}{(x+a)(x+b)\ln^2(x+b)}$$

Now, $f(x)$ is defined for values of x, given by $x+a>0$ and $x+b>0$ and $x+b\neq 1$

Thus, $f'(x)$ is positive $\forall\ x$ in the domain of $f(x)$ if

$\ln(x+b)^{x+b}>\ln(x+a)^{x+a}$

i.e., $(x+b)^{x+b}>(x+a)^{x+a}$ [ln x is an increasing function]

which is always true if $b>a$.

138. (c) : We have, $f'(x)>0$ and $g'(x)<0\ \forall\ x\in R$

$f(x)$ is increasing and $g(x)$ is decreasing $\forall\ x\in R$

Hence, we have

$g(x)>g(x+1)$ and $fog\,(x)>fog\,(x+1)$

$g(x)>g(x-1)$ and $fog\,(x)>fog\,(x-1)$

$f(x)>f(x+1)$ and $gof\,(x)>gof\,(x+1)$

$f(x)>f(x-1)$ and $gof\,(x)>gof\,(x-1)$.

139. (a) : We have

$$yy'=x\left(\frac{y^2}{x^2}+\frac{\phi(y^2/x^2)}{\phi'(y^2/x^2)}\right)$$

$$\Rightarrow\ \frac{y}{x}\frac{dy}{dx}=\left(\frac{y}{x}\right)^2+\frac{\phi((y/x)^2)}{\phi'((y/x)^2)}$$

it is homogeneous equation.

Let $y=vx\quad\therefore\ \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\Rightarrow\ v\left\{v+x\frac{dv}{dx}\right\}=v^2+\frac{\phi(v^2)}{\phi'(v^2)}$$

$$\Rightarrow\ vx\frac{dv}{dx}=\frac{\phi(v^2)}{\phi'(v^2)}\quad\Rightarrow\ \frac{dx}{xv\,dv}=\frac{\phi'(v^2)}{\phi(v^2)}$$

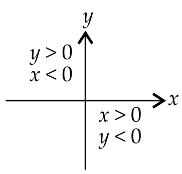
$$\Rightarrow\ \frac{2dx}{x}=\frac{2v\phi'(v^2)dv}{\phi(v^2)}$$

Integrating both sides we get

$2\log x=\log(\phi(v^2))-\log c\ \Rightarrow cx^2=\phi(v^2)$

$$\Rightarrow\ cx^2=\phi\left(\frac{y^2}{x^2}\right)$$

140. (b) : As branches lie in the second and fourth quadrant



$\therefore$ We have $xy<0$

$\Rightarrow\ m^2-9<0$

$\Rightarrow\ |m|<3$

141. (a) : $x-c=ay^2+by$

$$\Rightarrow\ x-c=\frac{a(ay^2+by)}{a}\quad\Rightarrow\ a\left(y^2+\frac{b}{a}y\right)=x-c$$

$$\Rightarrow\ a\left\{\left(y+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}\right\}=x-c$$

$$\Rightarrow\ a\left(y+\frac{b}{2a}\right)^2=\left(x+\frac{b^2}{4a}-c\right)$$

$$\Rightarrow\ \left(y+\frac{b}{2a}\right)^2=\frac{1}{a}\left(x+\frac{b^2-4ac}{4a}\right)$$

$$\Rightarrow\ Y^2=4\left(\frac{1}{4a}\right)X$$

$\Rightarrow$ length of latus rectum $=\frac{4}{4a}=\frac{1}{a}$.

142. (c) : Comparing the given equation

$ax^2+2hxy+by^2+2gx+2fy+c=0$

we have $a=1,h=0,b=4,g=1,f=8,c=13$

$$\text{Now }\Delta=\begin{vmatrix}a&h&g\\h&b&f\\g&f&c\end{vmatrix}=\begin{vmatrix}1&0&1\\0&4&8\\1&8&13\end{vmatrix}=-16\neq 0$$

$\Rightarrow$ given equation does not represent a pair of straight lines.

Again $h^2=0$ and $ab=4\ \Rightarrow\ h^2<ab$

Now if $\Delta\neq 0$ and $h^2<ab$, then second degree equation in x and y always represent an ellipse.

143. (c) : Given determinant shows the volume of parallelopiped formed by vectors $\vec a,\vec b,\vec c$

$$\therefore\ \begin{vmatrix}\alpha_1&\alpha_2&\alpha_3\\\beta_1&\beta_2&\beta_3\\\gamma_1&\gamma_2&\gamma_3\end{vmatrix}=|\vec a\cdot(\vec b\times\vec c)|$$

$$=|\vec a||\vec b\times\vec c|\cos\frac{\pi}{6}|=|\vec a||\vec b||\vec c|\cos\frac{\pi}{6}\cdot\sin\frac{\pi}{6}|$$

$$=2\sqrt2\,|\vec b||\vec c|\frac{\sqrt3}{2}\cdot\frac12=\frac{\sqrt3\,|\vec b||\vec c|}{\sqrt2}$$

$$\therefore\ \begin{vmatrix}\alpha_1&\alpha_2&\alpha_3\\\beta_1&\beta_2&\beta_3\\\gamma_1&\gamma_2&\gamma_3\end{vmatrix}^n=\left(\frac{\sqrt3\,|\vec b||\vec c|}{\sqrt2}\right)^n$$

144. (d) : Let $\vec r=a\hat i+b\hat j+c\hat k$

$\therefore\ \vec r\cdot\hat i=a,\ \vec r\times\hat i=-b\hat k+c\hat j$

$\therefore\ (\vec r\cdot\hat i)(\vec r\times\hat i)=ac\hat j-ab\hat k$ ...(i)

$(\vec r\cdot\hat j)(\vec r\times\hat j)=ab\hat k-bc\hat i$ ...(ii)

$(\vec r\cdot\hat k)(\vec r\times\hat k)=-ac\hat j+bc\hat i$ ...(iii)

On adding (i), (ii) & (iii) we get

$$(\vec r\cdot\hat i)(\vec r\times\hat i)+(\vec r\cdot\hat j)(\vec r\times\hat j)+(\vec r\cdot\hat k)(\vec r\times\hat k)=\vec 0$$

145. (c) : $\sin x+\cos x+\tan x+\cot x+\sec x+\mathrm{cosec}\,x=7$

$$\Rightarrow\ (\sin x+\cos x)+\frac{1}{\sin x\cos x}+\frac{\sin x+\cos x}{\sin x\cos x}=7$$

$$\Rightarrow\ (\sin x+\cos x)\left(1+\frac{2}{\sin 2x}\right)=\left(7-\frac{2}{\sin 2x}\right)$$

Squaring both sides

$$(1+\sin 2x)\left(1+\frac{4}{\sin^2 2x}+\frac{4}{\sin 2x}\right)=49+\frac{4}{\sin^2 2x}-\frac{28}{\sin 2x}$$

$\Rightarrow\ \sin^3 2x-44\sin^2 2x+36\sin 2x=0$

$\Rightarrow\ \sin 2x=22-8\sqrt7\quad\Rightarrow\ m=22,\ n=8$

146. (d): $\sin^2 x + \sin^2 y - \sin^2 z$

$= \sin^2 x + \sin(y+z)\sin(y-z)$

$= \sin^2 x + \sin(y+z)\sin(\pi - x)$

$= \sin x\,[\sin x + \sin(y+z)]$

$= \sin x\,[\sin(\pi + z - y) + \sin(y+z)]$

$= \sin x\,[\sin(y+z) - \sin(z-y)]$

$= 2\sin x \cos z \sin y$

147. (c) : Let $R = \{(x, y) : x + 2y = 8, x, y \in N\}$

$\therefore\ x + 2y = 8$ (which must be a natural number)

$\Rightarrow\ y = \dfrac{8-x}{2} \quad \therefore\ x = \{2, 4, 6\} \quad \therefore\ y = \{3, 2, 1\}$

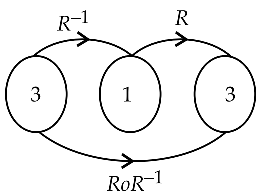
$\therefore\ R = \{(x, y) : x + 2y = 8\}$

$\Rightarrow\ R = \{(2, 3), (4, 2), (6, 1)\}\ \therefore$ Range of $R = \{1, 2, 3\}$

148. (b) : $A = \{1, 2, 3, 4\}, B = \{1, 3, 5\}$

$R = \{(a, b) : a < b, a, b \in A\}$

$= \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$



$\therefore\ R^{-1} = \{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$

Now $(3, 1) \in R^{-1}, (1, 3) \in R$

$\therefore\ (3, 3) \in RoR^{-1}$.

$(3, 1) \in R^{-1},\ (1, 5) \in R \Rightarrow (3, 5) \in RoR^{-1}$

$(5, 1) \in R^{-1},\ (1, 3) \in R \Rightarrow (5, 3) \in RoR^{-1}$

$(3, 2) \in R^{-1},\ (2, 3) \in R \Rightarrow (3, 3) \in RoR^{-1}$

$(3, 2) \in R^{-1},\ (2, 5) \in R \Rightarrow (3, 5) \in RoR^{-1}$

$(5, 2) \in R^{-1},\ (2, 3) \in R \Rightarrow (5, 3) \in RoR^{-1}$

$(5, 2) \in R^{-1},\ (2, 5) \in R \Rightarrow (5, 5) \in RoR^{-1}$

$(5, 3) \in R^{-1},\ (3, 5) \in R \Rightarrow (5, 5) \in RoR^{-1}$

$(5, 4) \in R^{-1},\ (4, 5) \in R \Rightarrow (5, 5) \in RoR^{-1}$

$\therefore\ RoR^{-1} = \{(3, 3), (3, 5), (5, 3), (5, 5)\}$.

149. (c) : Making the equation of the curve homogeneous with the help of the line, we get

$$x^2 + y^2 - 2(kx + hy)\left(\frac{hx+ky}{2hk}\right) \quad \left[\because \frac{hx+ky}{2hk} = 1\right]$$

$$+\ (h^2 + k^2 - c^2)\left(\frac{hx+ky}{2hk}\right)^2 = 0$$

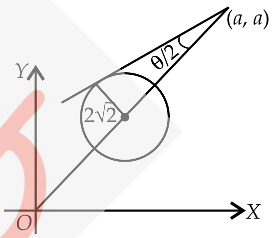
or $4h^2k^2x^2 + 4h^2k^2y^2 - 4hk^2x(hx+ky) - 4h^2ky(hx+ky) + (h^2+k^2-c^2)(h^2x^2 + k^2y^2 + 2hkxy) = 0$

This is the equation of the pair of lines joining the origin to the points of intersection of the given line and the curve. They will be at right angles if coefficient of x^2 + coefficient of $y^2 = 0$.

$(h^2 + k^2)(h^2 + k^2 - c^2) = 0$ since $[h^2 + k^2 \neq 0]$

$\Rightarrow\ h^2 + k^2 = c^2$.

150. (d): The centre of the circle is (1, 1) and radius $= 2\sqrt{2}$.



From (a, a) must lie outside the circle, so

$2a^2 - 4a - 6 > 0 \Rightarrow a < -1$ or $a > 3$

Now, $\tan\dfrac{\theta}{2} = \dfrac{2\sqrt{2}}{\sqrt{2a^2 - 4a - 6}}$.

As $\dfrac{\pi}{3} < \theta < \pi \Rightarrow \dfrac{\pi}{6} < \dfrac{\theta}{2} < \dfrac{\pi}{2}$

$\therefore\ \dfrac{2\sqrt{2}}{\sqrt{2a^2 - 4a - 6}} > \dfrac{1}{\sqrt{3}} \Rightarrow \sqrt{a^2 - 2a - 3} < 2\sqrt{3}$

$\therefore\ a^2 - 2a - 15 < 0 \Rightarrow -3 < a < 5$

$\therefore\ a \in (-3, -1) \cup (3, 5)$.